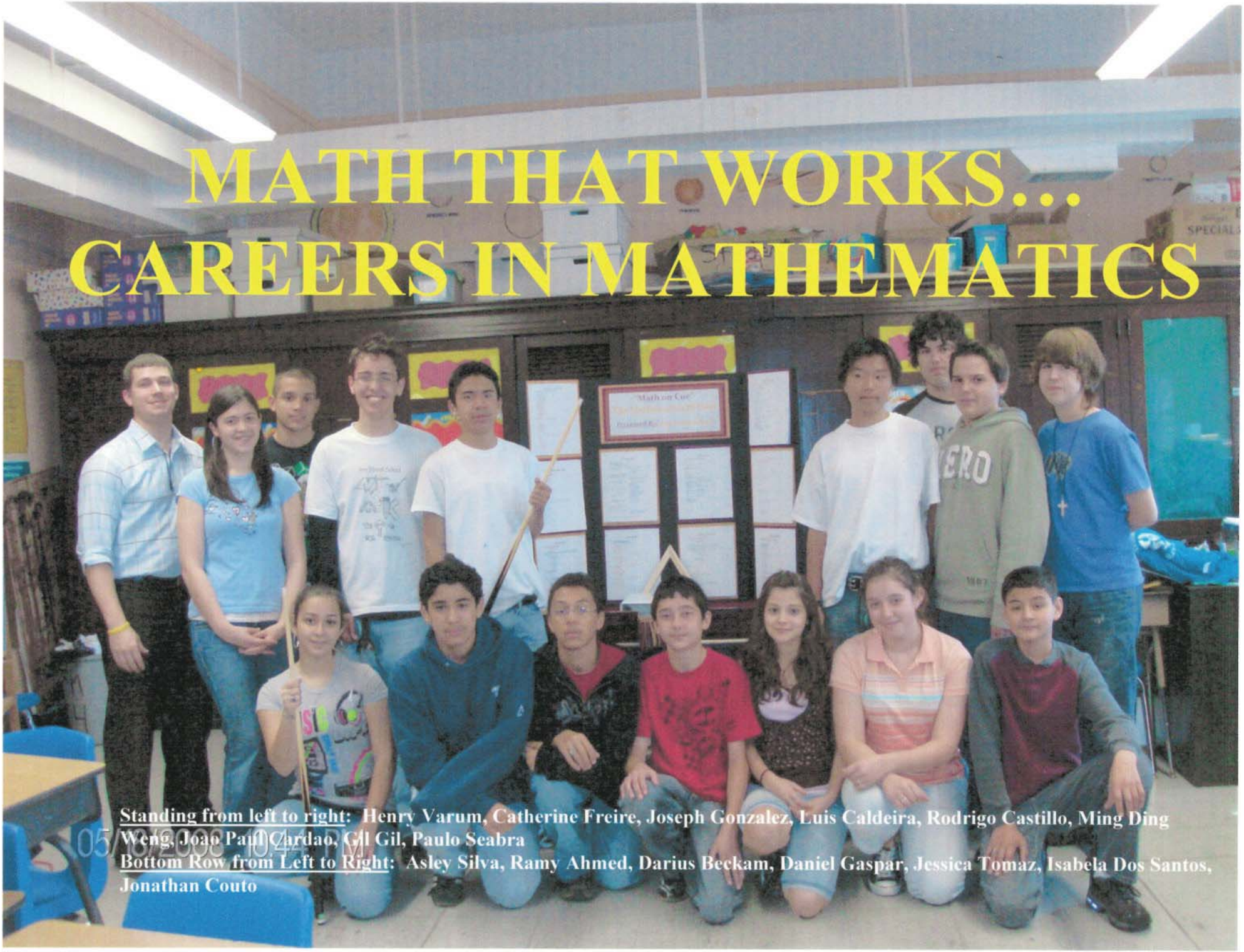
MATH THAT WORKS…
CAREERS IN MATHEMATICS



Standing from left to right: Henry Varum, Catherine Freire, Joseph Gonzalez, Luis Caldeira, Rodrigo Castillo, Ming Ding Weng, Joao Paul Cardao, Gil Gil, Paulo Seabra
Bottom Row from Left to Right: Asley Silva, Ramy Ahmed, Darius Beckam, Daniel Gaspar, Jessica Tomaz, Isabela Dos Santos, Jonathan Couto

Math that Works...Careers in Mathematics
"Math on Cue"
The Mathematics of Billiards

"POOL TABLE!" Luis Caldeira screamed out. Everyone became dead silent. Minutes before, our Algebra class was debating ideas for our math fair project. We wanted something new, something fresh. Therefore, the average architecture project was out of the question. There was a glint of mischievousness in Mr. Varum's eyes. We had our project.

At first glance, the idea of a professional pool player as a mathematical career seemed far fetched. However, once we started analyzing all of the information we had collected, we realized that a professional pool player consciously performs more mathematical calculations than we could have imagined.

Mr. Varum then took this project one step further and surprised us with a miniature billiard ball. We were now building a table and the career of a carpenter became another part of our project. Isabela Dos Santos took on this job by scaling down a competition sized pool table so that it was proportional to the miniature ball.

Our math work consists of sine, cosine and tangent ratios as well as the use of the Pythagorean Theorem. We used our understanding of angles, conversion and scaling of measurements, and proportions in order to accurately portray all of our mathematical data.

We have also made diagrams of our shots to explain each step of our work in great detail. While we were working on our shots, we were also building our pool table. During the process, we had to cut, glue, and hammer various pieces of wood together.

We measured angles and side lengths, varnished and stained wood, as well as create all the pockets for the table.

This elaborate and intricate project took countless hours of work, patience, and effort. When everything was complete, our Algebra class and Mr. Varum stood proud as we observed our finished work of art.

Scaled Measurements of Pool Table

Pool Table Measurements

Scale Factor

Actual ball diameter – 2.25 in.
Model ball diameter – 1.375
-Scale factor from actual to model $\frac{1.375}{2.25} = \frac{11}{18}$

Actual	Model
Table- **4.5 by 9 ft**	$4.5 \times (11/18) =$ **2.75ft.** ↑ Scale factor from actual to model. $9 \times (11/18) =$ **5.5 ft.**
Rail height must be over ½ of the diameter of the ball but less than 64.5% of the diameter of the ball	Diameter of ball for model. ↓ $1.375 \times .645 = .886875$ in. –maximum height $1.375 \div 2 = .6875$ in. – minimum height We used **.75 in.** for our model.
Rail width- **4 - 7 ½** inches with cushions	$4 \times (11/18) =$ **2.444 in. minimum** $7.5 \times (11/18) =$ **4.58333 in. maximum** Rail-bumper-1.25 in. Wood rail-1.75 in. Our model's width is 3 in.
18 sights- diamonds **12 ½** inches apart	$12.5 \times (11/18) =$ **7.638888889 in.** $\approx$ **7.639 in. apart**
Corner pocket mouth- **4.5- 4.625** inches	$4.5 \times (11/18) = 2.75$ in. $4.625 \times (11/18) = 2.826388889$ in. $\approx$ **2.75- 2.8264 we used 2.75 in.**
Side mouth- **5 and 5.125** inches	$5.125 \times (11/18) = 3.13944444$ in. $5 \times (11/18) = 3.05$ in. **3.055 and 3.13944444** **We used 3.125 in.**
Cue sticks- minimum of 40 inches there is no maximum	$40 \times (11/18) = 24.4444$ in. our cue stick is **40 in.**
Cut angles of corner pockets = 142°	
Cut angles of side =104°	
Cloth- **4.5 × 9 = 40. 5** square feet of cloth	$40.5 \times (11/18)^2 =$ **24.75 square feet of cloth needed for the playing surface of the model**

Pockets- usually rimmed with leather or plastic (pockets or ball returns)

Measurements for Table Diagram

Scale factor from actual table to diagram-
1in. on diagram = 1ft. on actual table
Scale factor = **1/12**

Scale factor from model table to diagram
5.5 x 12 = **66in.**
Model = **9in.**

Scale factor = **9/66 = 3/22**

Scaled Measurements

Diagram measurements

Dimensions – 4.5ft. x 9ft.
5.5ft. x 3/22 = .75f. x 12 = **9in.**
2.75ft. x 3/22 = .6136ft. = .375 x 12 = **4.5in.**

Rail Cushion
1.25in. x 3/22 = **.17045in.**
1.75in. x 3/22 = **.238636in.**

Sights
7.639in. x 3/22 = **1.041681in.**

Corner Pockets
2.75in. x 3/22 = **.375in.**

Side Pockets
3.125in. x 3/22 = **.426136in.**

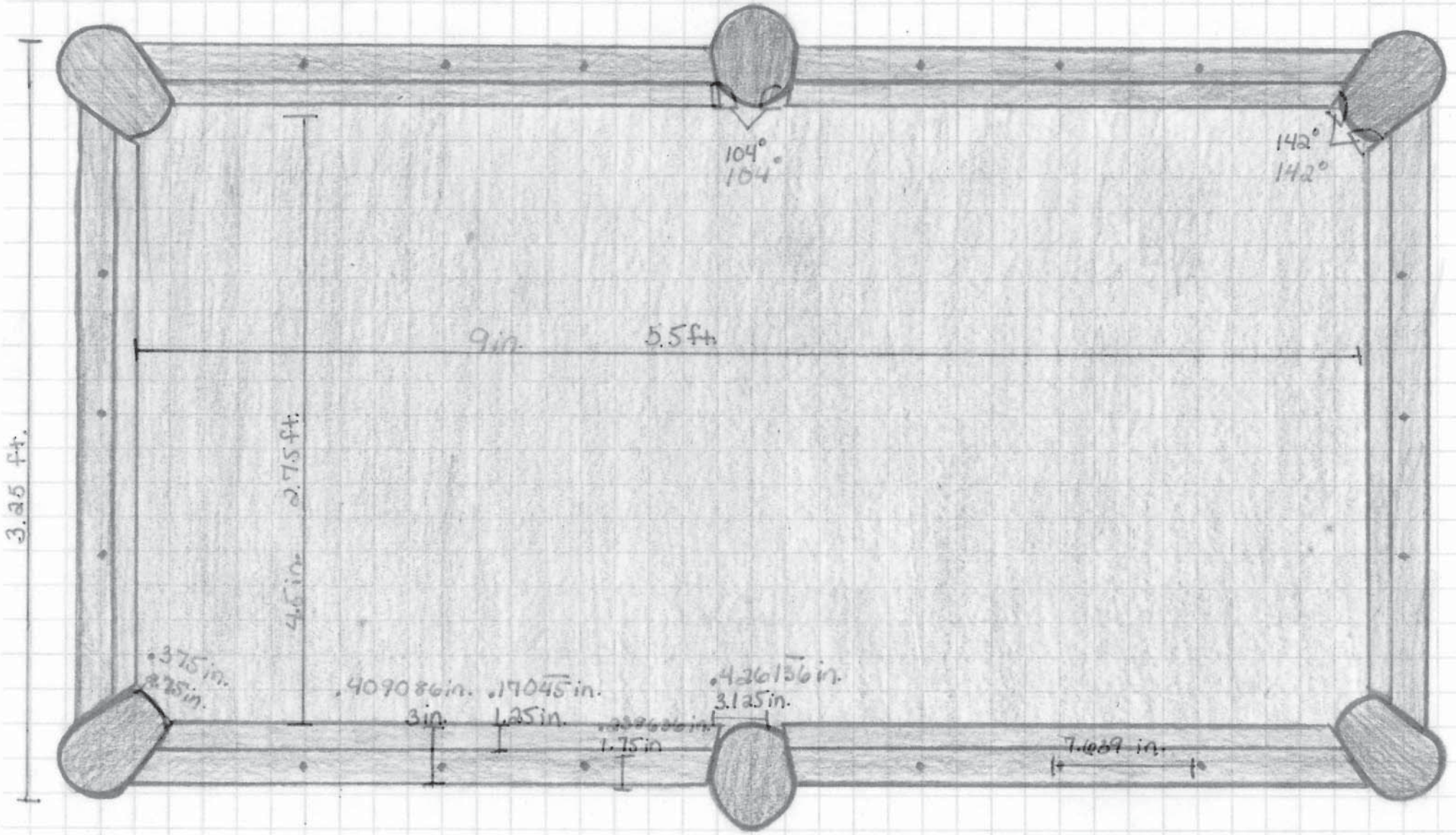
Cut angle of corner pocket
142°

Cut angle of side pocket
104°

Diagram of Model

6 ft.

scale factor - $\frac{3}{22}$ 3in. = 22 in.



Measurements of the Diagram based on Actual Pool Table Lengths

scale factor - $\frac{1}{12}$ 1in. = 12 in.

Jessica's Angle Shot

Jessica's Angle Shot

Angle at Which Target Ball Travels

(5,1)
a= 5ft. - 4.5 ft. a = .5 ft.
b = 1 ft.
Tan-1(1/.5) = 63.43494882° = 63.43°
63.43°+ 90° = 153.43°
180°- 153.43° = **26.57°**

Measure of a` and b`

Cos63.43° = a`/2.25
.4472908484 = a`/2.25
1.006404490 = a` **1in. = a`**
Sin63.43° = b`/2.25
.8943885604 = b`/ 2.25
2.012374261= b` **2.01= b`**

Center of Target Ball in Feet

(5,1)

Center of Target Ball in Inches

5×12 = 60 in.
1×12 = 12 in. **(60,12 in.)**

Center of Cue Ball at Contact

60 in.+1 in. = 61 in. (x)
12 in.+2.01 = 14.01 in. (y)
Convert **(61,14.01 in.)** to feet
61 in.÷12 in. = 5.08 ft.
14.01 in.÷12 in. = 1.1675 ft.
(5.08,1.1675 ft.)

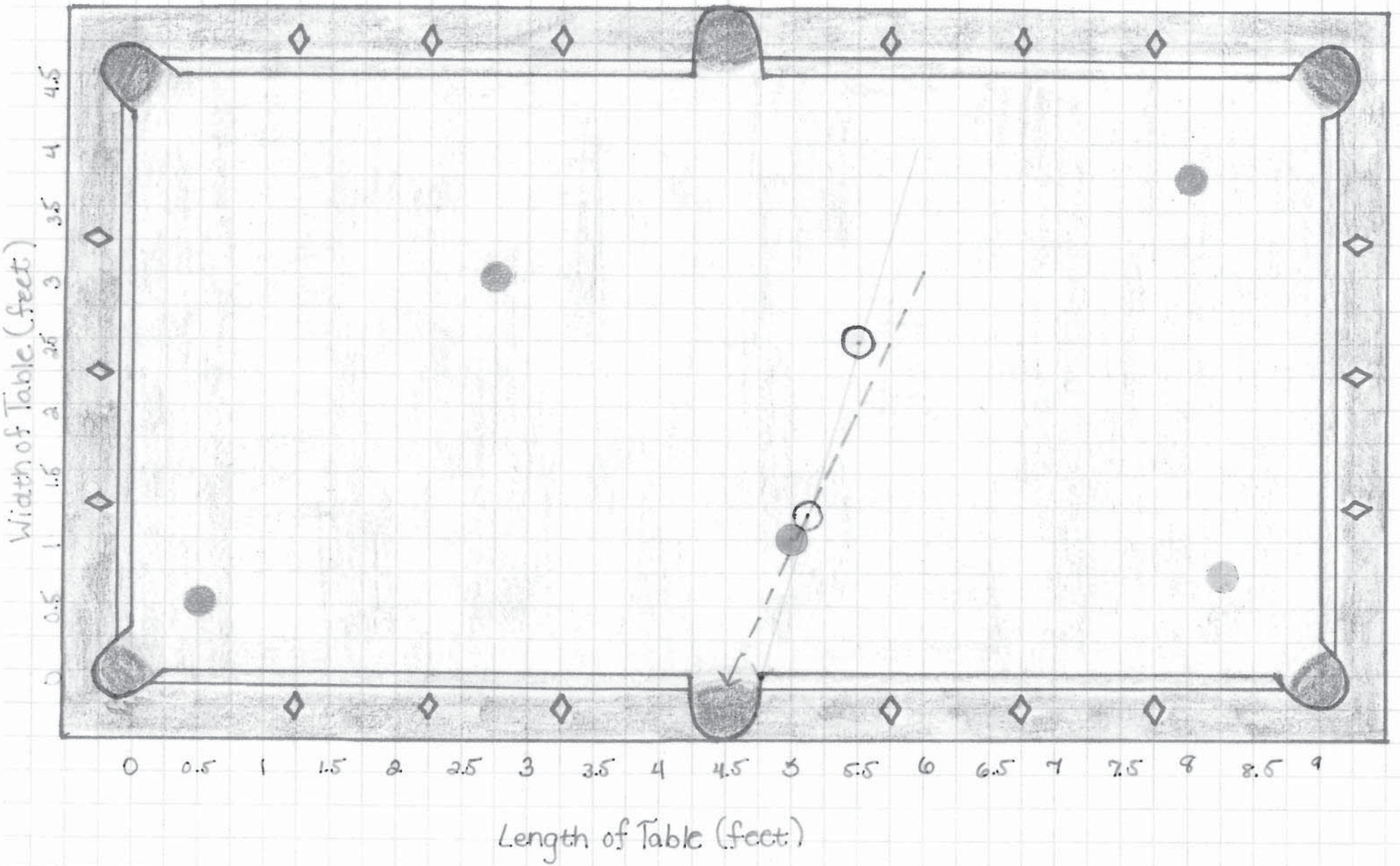
Point of Contact Between Cue Ball and Target Ball

2.01÷2 = 1.005 in.
1÷2 = .5 in.

12+1.005 = 13.005 in.
60+.5 = 60.5 in.
Convert **(60.5,13.005 in.)** to feet
60.5 in.÷12 in.= 5.04 ft.
13.005 in.÷12 in. = 1.08 ft.
(5.04,1.08 ft.)

Angle Cue Ball Travels At

5.5 ft.-5.08ft. = .42 ft.
2.5 ft.-1.1675 ft. = 1.3325 ft.
Tan-1(1.3325/.42) = 72.50530497 ° = 72.5 °
180 °-72.5 °-90 ° = **17.5** °


Width of Table (feet)
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Length of Table (feet)
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0.5
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2.5
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3.5
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4.5
5
5.5
6
6.5
7
7.5
8
8.5
9

Angle at Which Target Ball Travels

(5,1)

$a = 5\text{ft.} - 4.5\text{ft.}$ $a = .5\text{ft}$

$b = 1\text{ft.}$

$\text{Tan}^{-1}\left(\frac{1}{.5}\right) = 63.43494882^\circ = 63.43^\circ$

$63.43^\circ + 90^\circ = 153.43^\circ$

$180^\circ - 153^\circ = 26.57$

Measure of a' and b'

$\text{Cos}\,63.43^\circ = \frac{a'}{2.25}$

$.4472908454 = \frac{a'}{2.25}$

$1.006404490 = a'$ $1\text{in.} = a'$

$\text{Sin}\,63.43^\circ = \frac{b'}{2.25}$

$.8943885604 = \frac{b'}{2.25}$ (5.04, 1.08) (60.5, 13.005)

$2.012374261 = b'$ $2.01 = b'$

Center of Target Ball in Feet.

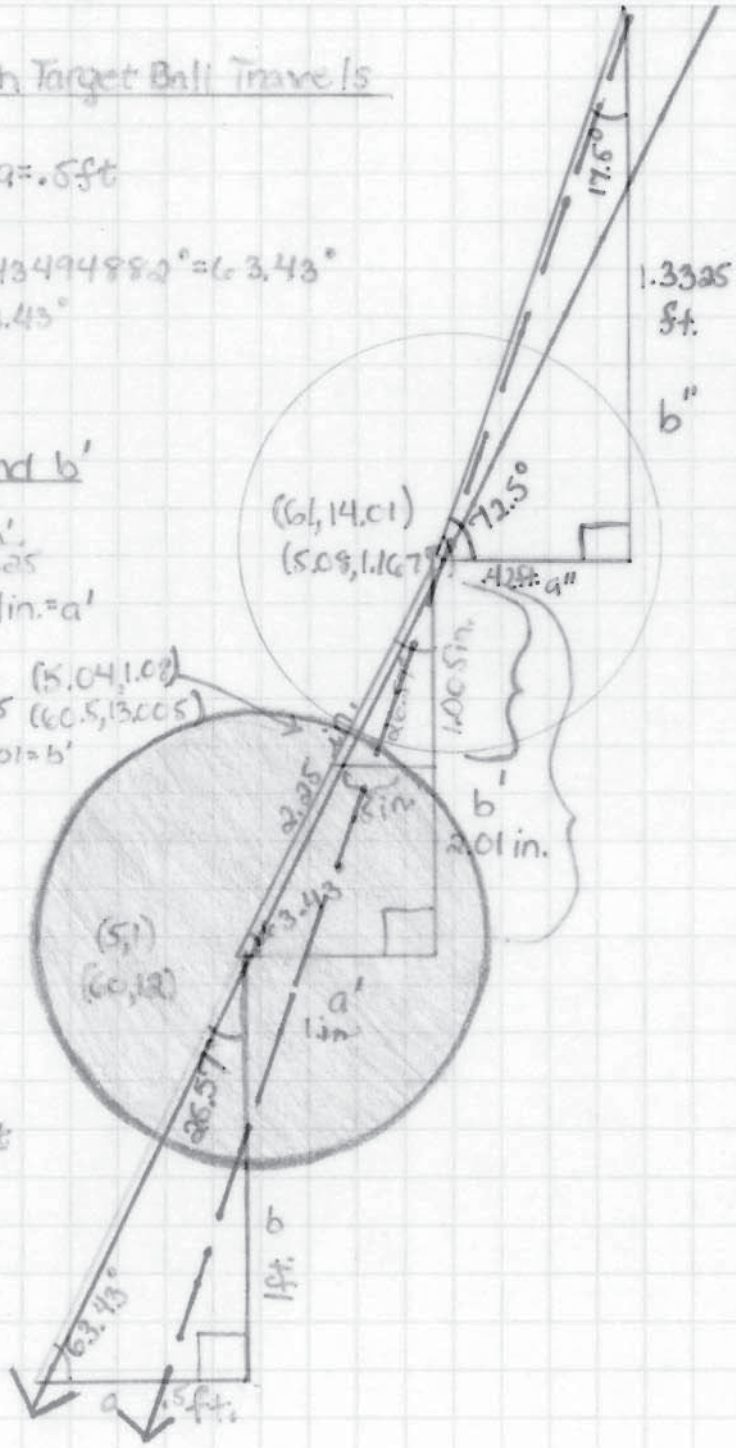
5,1)

Center of Target Ball in Inches

$5 \times 12 = 60\text{ in.}$

$1 \times 12 = 12\text{ in.}$

(60, 12 in.)



Center of Cue Ball at Contact

$60\text{ in.} + 1\text{ in.} = 61\text{in.}(x)$

$12\text{ in.} + 2.01 = 14.01\text{ in.}(y)$

Convert (61, 14.01 in.) to feet

$61\text{in.} \div 12\text{ in.} = 5.08\text{ ft.}$

$14.01 \div 12\text{ in.} = 1.1675\text{ ft.}$

(5.08, 1.1675 ft.)

Point of Contact Between Cue Ball and Target Ball

$2.01 \div 2 = 1.005\text{ in.}$

$1 \div 2 = .5\text{in.}$

$12 + 1.005 = 13.005\text{in.}$

$60 + .5 = 60.5\text{in.}$

Convert (60.5, 13.005) to feet

$60.5\text{in.} \div 12\text{in} = 5.04\text{ft.}$

$13.005\text{in} \div 12\text{in} = 1.08\text{ft.}$

(5.04, 1.08 ft.)

Angle Cue Ball Travels At.

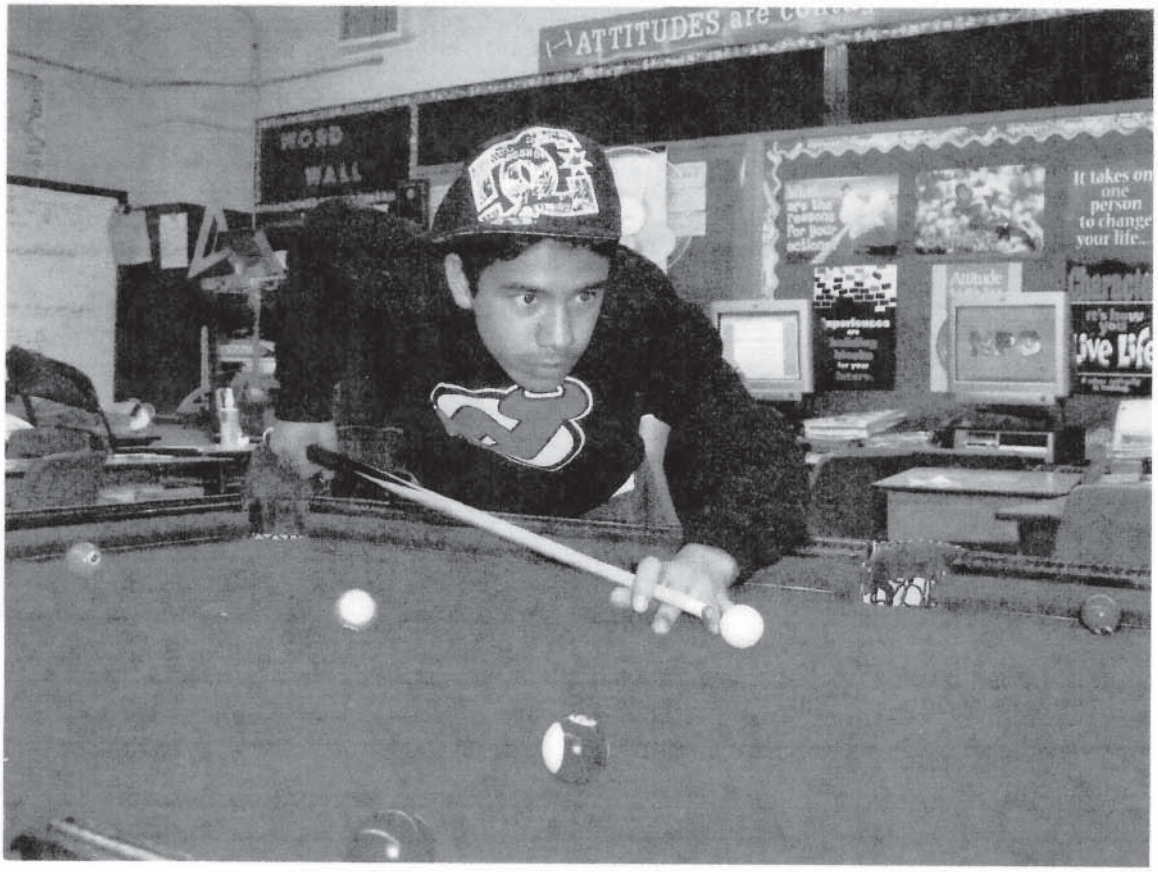
$5.5\text{ft.} - 5.08\text{ft.} = .42\text{ ft.}$

$2.5\text{ft.} - 1.1675\text{ ft.} = 1.3325\text{ ft.}$

$\text{Tan}^{-1}\left(\frac{1.3325}{.42}\right) = 72.50530497^\circ = 72.5^\circ$

$180^\circ - 72.5^\circ - 90^\circ = 17.5^\circ$

Measurements may not be to scale



Rodrigo's Angle Shot

Rodrigo's Angle Shot

Angle Target Ball must be hit at

$TAN^{-1}\frac{1.25}{1.50} = 39.80557109^{\circ}$

$=39.8^{\circ}$

Center of Cue Ball At Contact

Side a Measurements

COS 39.8° = "/2.25

~~(2.25)"/2.25~~

.7682835236 * 2.25= 1.728637928

a= 1.73 in.

/ 12

a= .1441666667ft.

Side b Measurements

SIN 39.8° = "/2.25

~~(2.25)"/2.25~~

.6401096995 * 2.25= 1.440246824

b=1.44 in.

/12

b= .12 ft.

Center of Cue Ball at Contact

-Coordinates of Target Ball

(7.75, 1.5) ft.

*12

(93, 18) in.

93-1.44=91.56in.

18+1.73=19.73in.
(91.56, 19.73) in.
/12
(7.63, 1.644) ft.

Point of Contact Between Target and Cue Ball

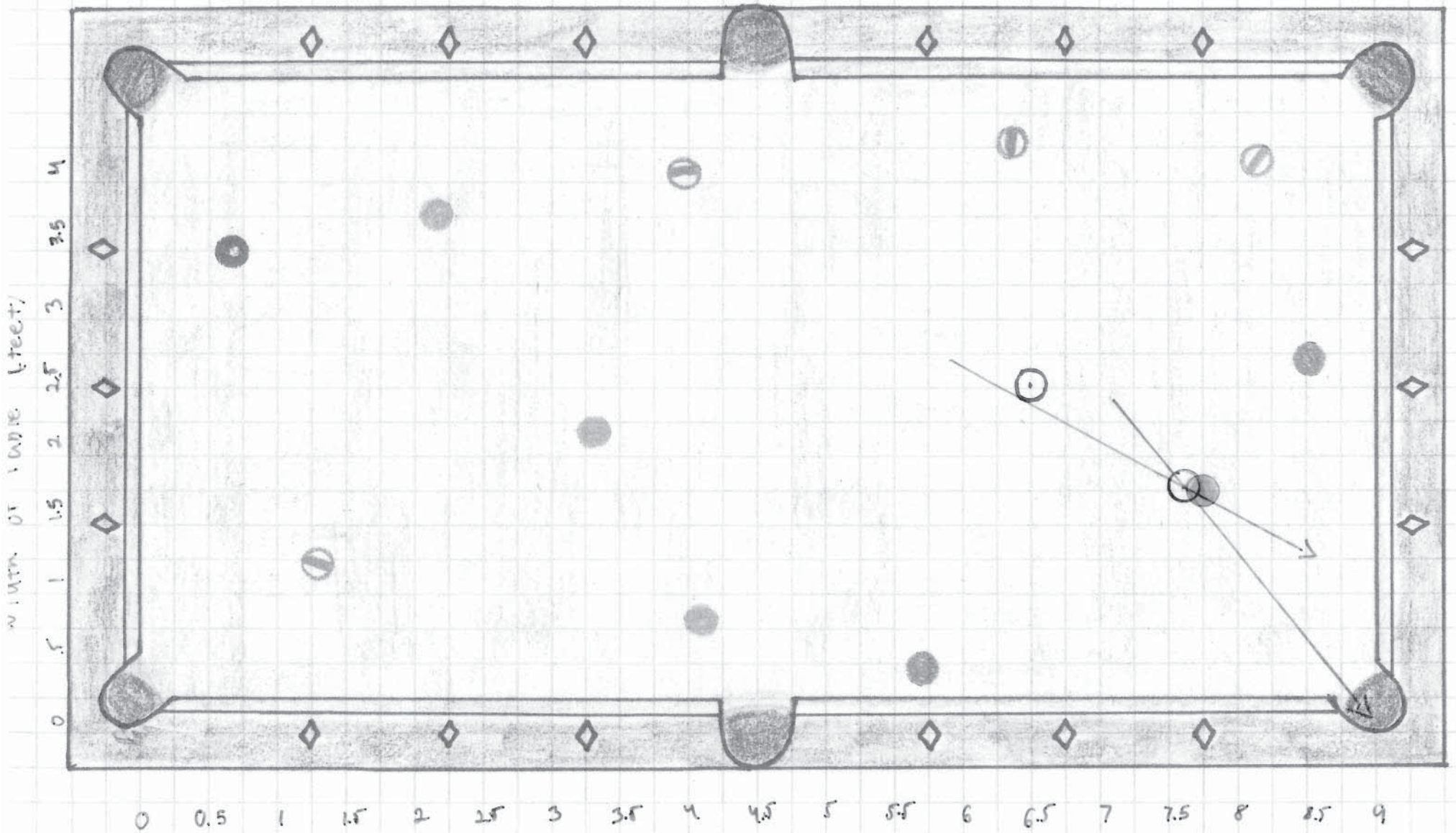
(measurement of side a)1.73/2=.865in.
(measurement of side b)1.44/2=.72in.
93-.72=92.28 in.
18+.865=18.865 in.
(92.28, 78.865) in.
/12
(7.69, 1.572083333) ft.

Angle Cue Ball is Struck At

7.63-6.5=1.13 ft.
Or *12
91.56-78=13.56in.

2.25-1.644=.606 ft.
Or *12
27-19.728=7.272in.

$TAN^{-1}(1.13/.606)$
61.7960889^{o}
$=61.8^{o}$


Width of Table (feet)
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4.5
5
5.5
6
6.5
7
7.5
8
8.5
9
Length of Table (feet)

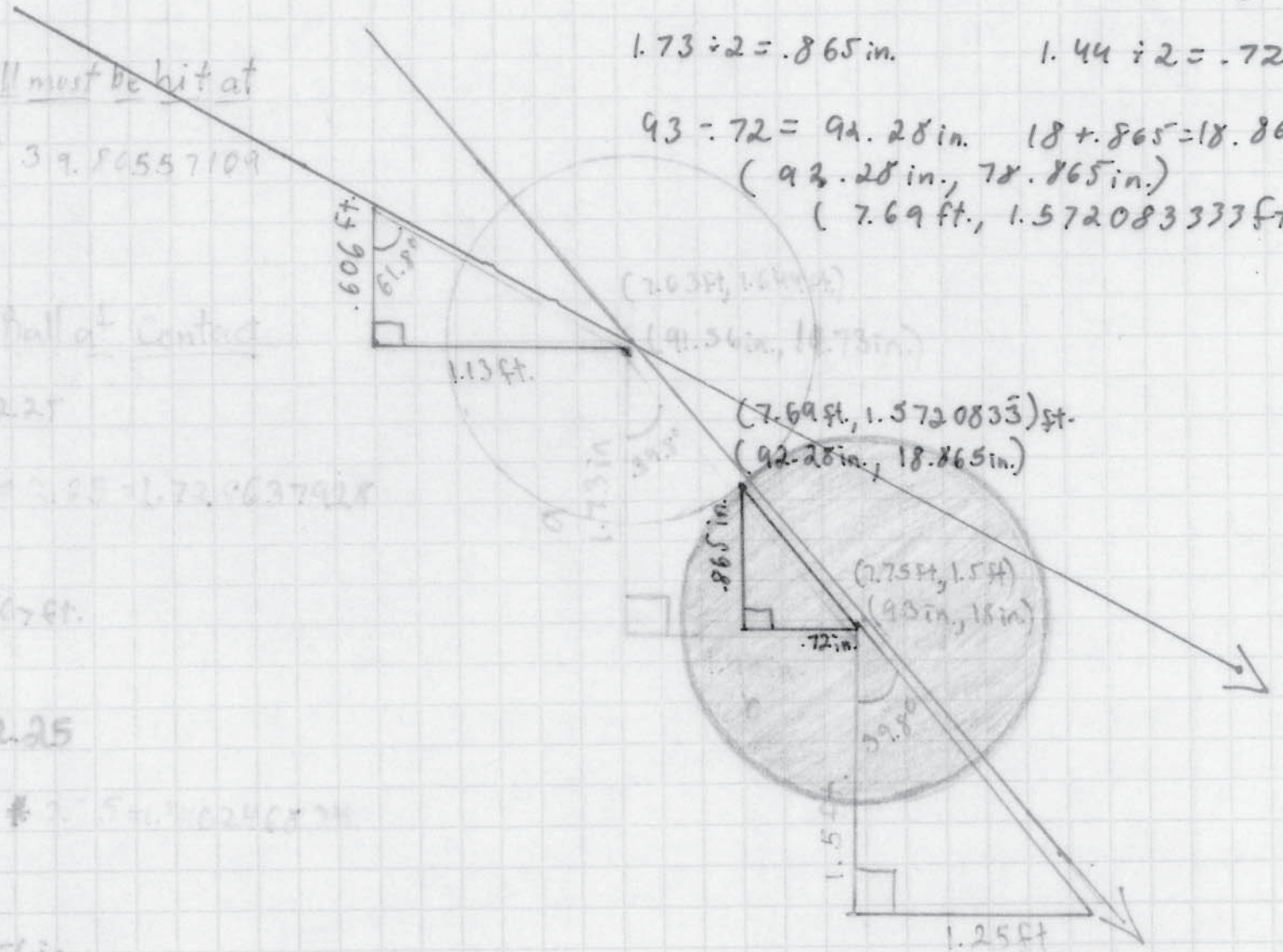
Point of Contact Between Cue Ball and Target Ball

$1.73 \div 2 = .865$ in. $\quad 1.44 \div 2 = .72$ in.

$93 - .72 = 92.28$ in. $\quad 18 + .865 = 18.865$ in.

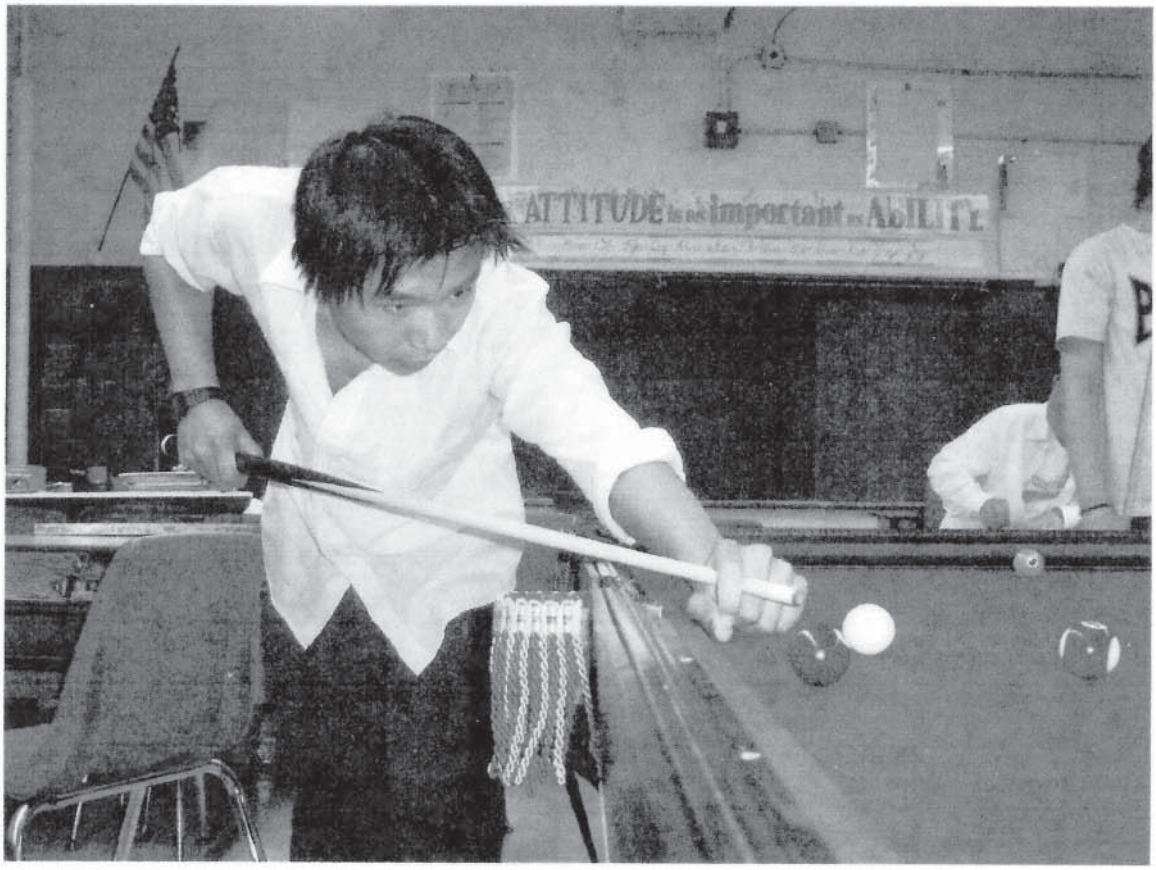
(92.28 in., 78.865 in.)

(7.69 ft., 1.572083333 ft.)



Angle Cue Ball Travels In

Measurements May Not Be to Scale



Ming's Angle Shot

Ming's Angle Shot

Angle Target Ball Travels At

$TAN^{-1}\frac{3}{1.75} = 59.74356284°$

$=59.74°$

Side a Measurement

$SIN\ 59.74 = \frac{a}{2.25}$

$.8637475669 = \frac{a}{2.25}$

$(2.25).8637475669 = \frac{a}{2.25}\ (2.25)$

$a=1.943432026$

$a=1.94$ in.

$1.94/12= .1616$

$a=.1616$ ft.

Side b Measurement

$COS\ 59.74 = \frac{b}{2.25}$

$.5039247371 = \frac{b}{2.25}$

$(2.25)\ .5039249371 = \frac{b}{2.25}\ (2.25)$

$b=1.133830658$

$b=1.13$ in

$1.13/12=.09417$

$b=.09417$ ft.

Coordinate of Center of Cue Ball (at contact with target ball)

(21, 18)

$21+1.13=22.13/12=1.84416$

$18-1.94=16.06/12=1.3303$

(1.84,1.33) ft

(21.113, 16.06) in.

Coordinate of Center of Target Ball

(1.75, 1.5)
1.75*12 = 21 in.
1.5*12 = 18 in.
(1.75, 1.5) ft.

1.75*12=21
1.5*12=18
(21, 18) in

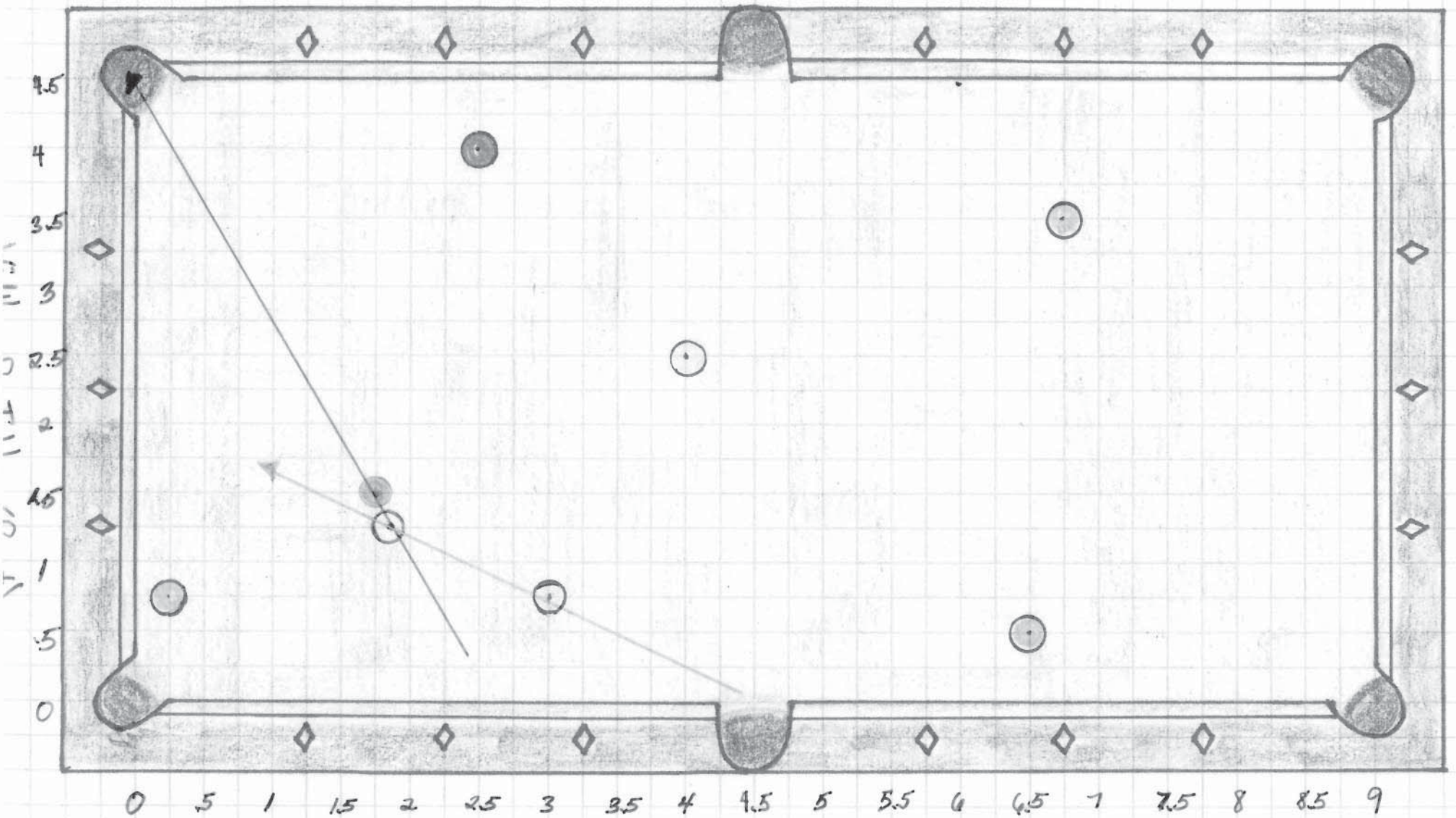
Point of Contact

$\frac{(1)}{2}$ 1.94 = .97
$\frac{(1)}{2}$ 1.13 = .565
21+.97 = 21.97/12 = 1.83083 ft.
18-.565 = 17.435/12 = 1.452916 ft.
(1.83083, 1.452916) ft.
(21.97, 17.435) in

Angle at Which Cue Ball is Struck

36-21.13=14.87
16.06-9=7.06

$TAN^{-1} \frac{7.06}{14.08} = 26.63012302°$
$= 26.63°$


4.5
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.5
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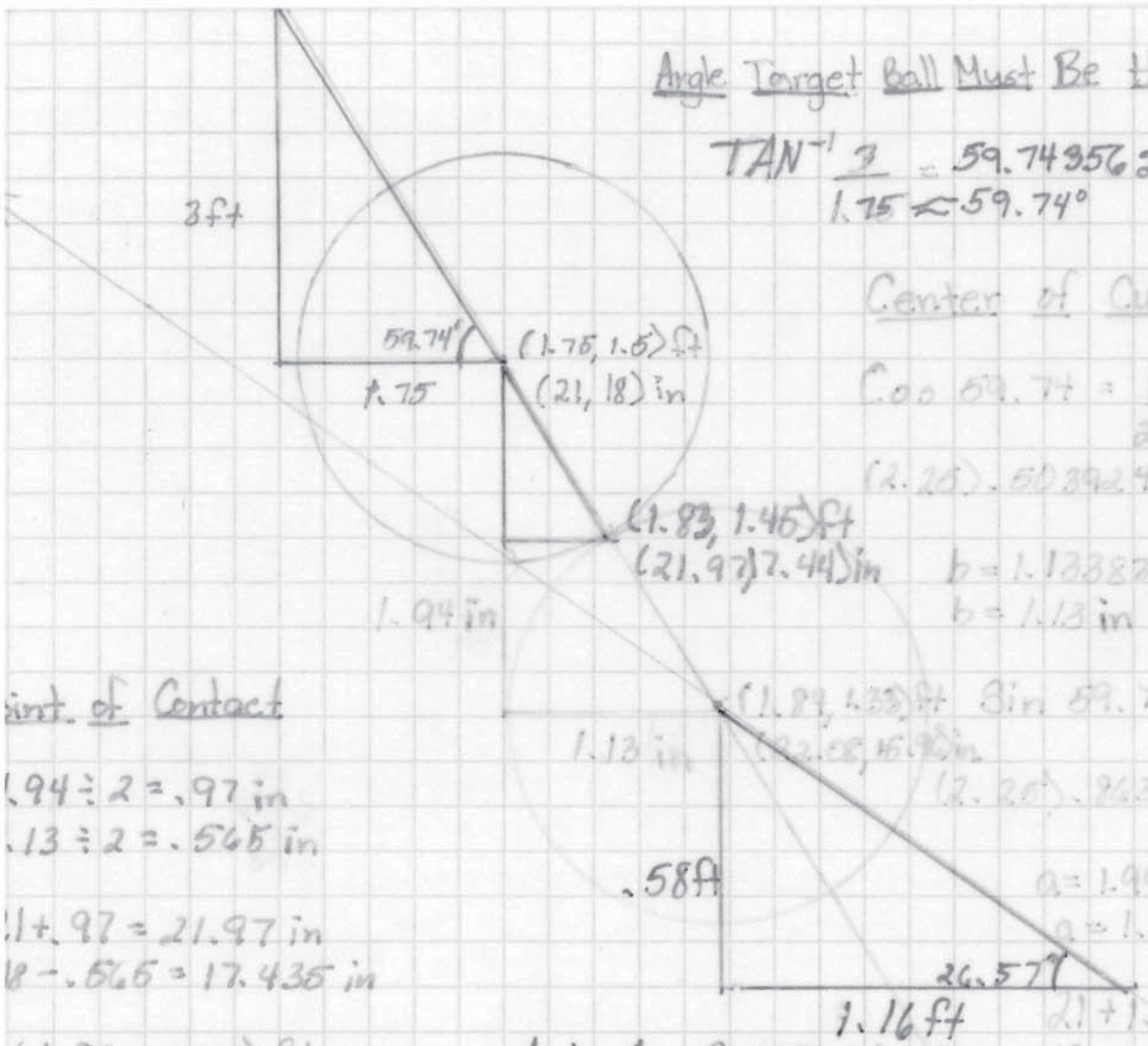
0 .5 1 1.5 2 2.5 3 3.5 4 4.5 5 5.5 6 6.5 7 7.5 8 8.5 9
Length of Table (feet)

Angle Target Ball Must Be Hit At

$\tan^{-1}\frac{3}{1.75} = 59.74356284°$

$\frac{3}{1.75} \approx 59.74°$



Center of Cue Ball at Contact

$\cos 59.74° = \frac{b}{2.25}$

$(2.25)(.5039249371) = b \quad (2.25)$

$b = 1.133820658$

$b = 1.13$ in

$\sin 59.74° = \frac{a}{2.25}$

$(2.25)(.8637475669) = a \quad (2.25)$

$a = 1.943432024$

$a = 1.94$ in

Point of Contact

$1.94 \div 2 = .97$ in

$1.13 \div 2 = .565$ in

$21 + .97 = 21.97$ in

$18 - .565 = 17.435$ in

(1.83, 1.45) ft

(21.97, 17.44) in

$21 + 1.13 = 22.13$

$18 - 1.94 = 16.06$

(1.84, 1.33) ft

(22.08, 15.96) in

Angle Cue Ball Travels At

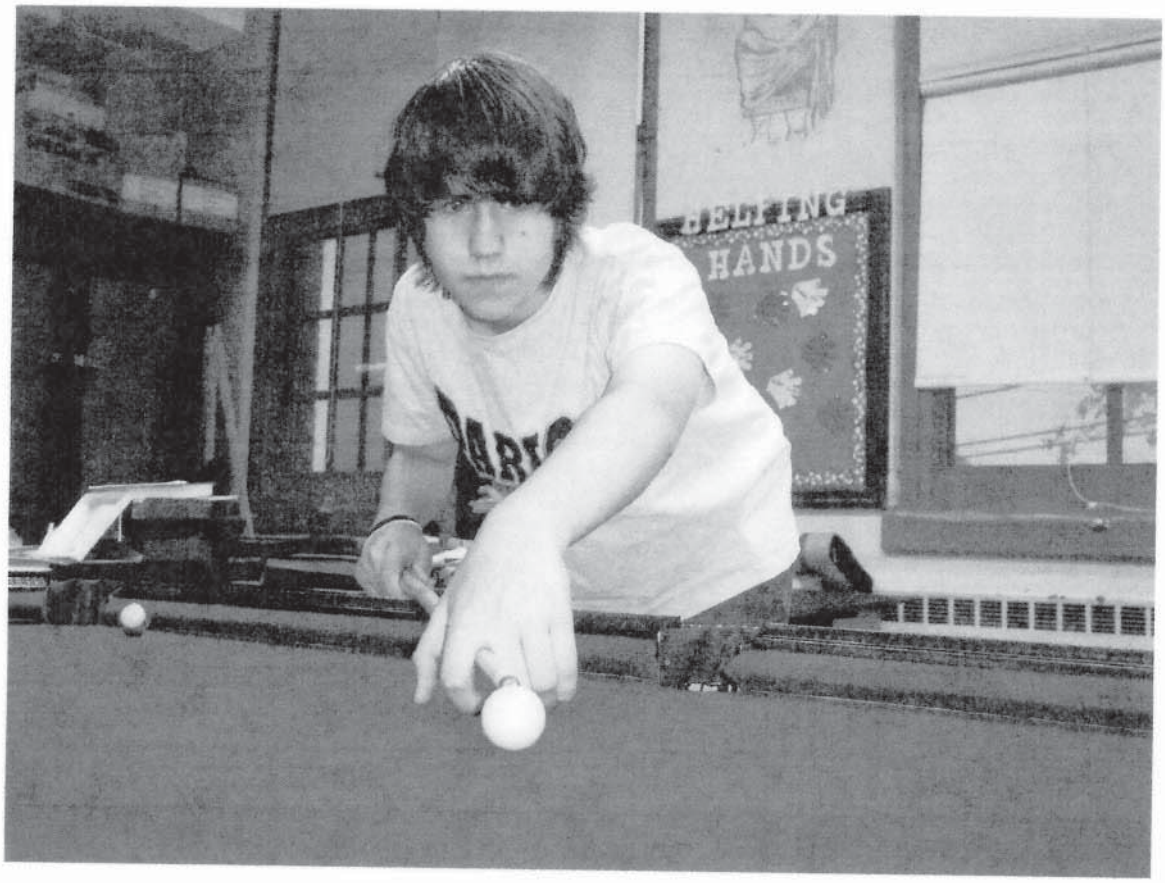
$36 - 21.13 = 14.87$

$16.06 - 9 = 7.06$

$\tan^{-1}\frac{7.06}{14.08} = 26.63012302°$

$\frac{7.06}{14.08} \approx 26.63°$

Measurements May NOT BE TO SCALE!



Paul's Angle Shot

Paulo's Angle Shot

Angle Target Tall Must Be Hit At

$\tan^{-1}(.75/.75) = 45°$

Center Of Cue Ball At Contact

Measure of a'

$\sin(45°) = x/2.25$

$(2.25)\ .707106781 = x/2.25\ (2.25)$

$1.590990258 = x$

$1.59 \text{ inches} = x$

Measure of b'

$\sin(45°) = x/2.25$

$(2.25)\ .707106781 = x/2.25\ (2.25)$

$1.590990258 = x$

$1.59 \text{ inches} = x$

Center of Cue ball at contact

$(.75, 3.75)\ *12$

$(9, 45)$ inches

$9 + 1.59 = 10.59$ inches

$45 - 1.59 = 43.41$ inches

Inches-(10.59,43.41) Feet-(.882, 3.62)

$76.75/12 = 6.39583$

$46.87/12 = 3.90583$

Point Of Contact Between Cue Ball And Target Ball

$1.59/2 = .795$

(10.59, 43.41)

$10.59 - .795 = 9.795$ inches

$43.41 + .795 = 44.205$ inches

Inches-(9.795, 44.205) Feet-(.816, 3.68)

$9.795/12 = .81625$

$44.205/12 = 3.68375$

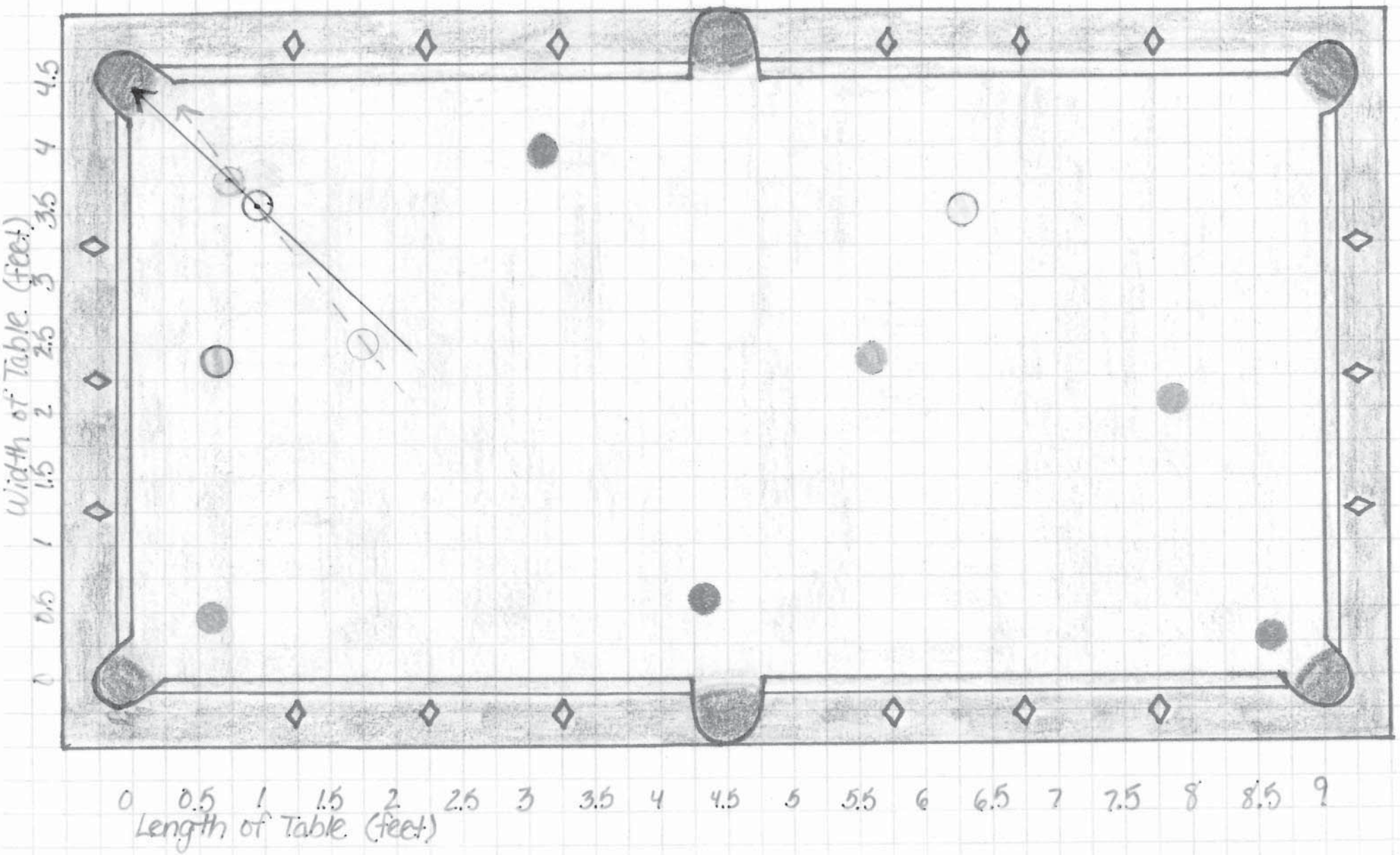
Angle Cue Ball is Shot At

18-9.795=8.205 inches

44.205-30=14.205 inches

$\tan^{-1}(14.205/8.205)=59.98868907°$

$\approx 59.99°$


Width of Table (feet)
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0.5
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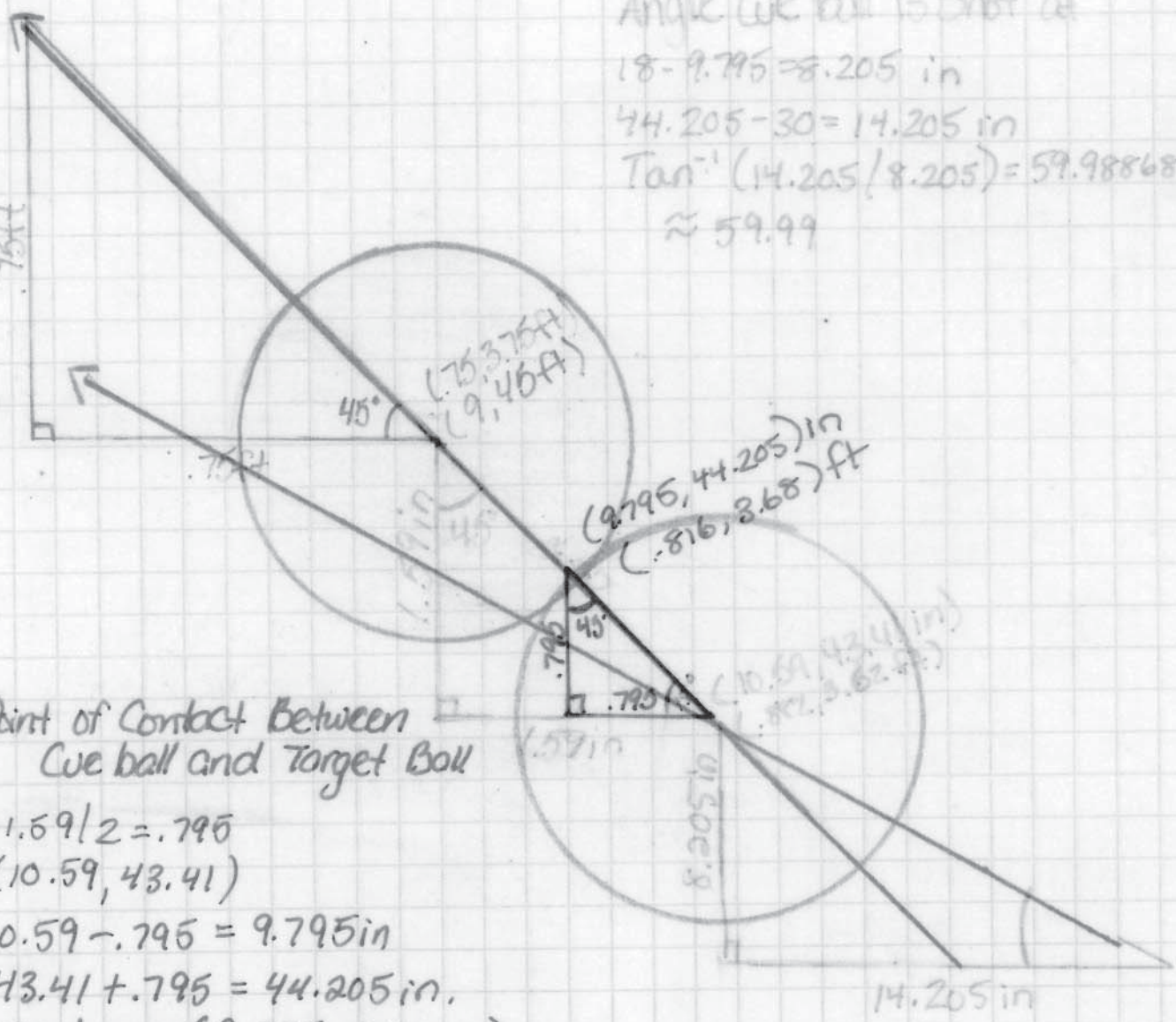
Length of Table (feet)
0 0.5 1 1.5 2 2.5 3 3.5 4 4.5 5 5.5 6 6.5 7 7.5 8 8.5 9



Point of Contact Between Cue ball and Target Ball

1.59/2 = .795

(10.59, 43.41)

10.59 − .795 = 9.795in

43.41 + .795 = 44.205 in.

Inches - (9.795, 44.205)

Feet (.816, 3.68)

9.795/12 = 8.1625

44.205/12 = 3.68375

Angle Cue Ball is shot at

18 − 9.795 = 8.205 in

44.205 − 30 = 14.205 in

$\tan^{-1}(14.205/8.205) = 59.98868907°$

≈ 59.99

Angle target Ball must be hit at

$\tan^{-1}(.75/.75) = 45°$

Center of Cue ball at contact

$\sin(45°) = x/2.25$ (2.25)

$.707106781 = x/2.25$ (2.25)

$1.590990258 = x$

≈ 1.59 inches $= x$

$\cos(45°) = 2.25$ (2.25)

$.707106781 = x/2.25$ (2.25)

1.590990258

≈ 1.59 inches $= x$

$(.75, 3.75)$ ×12

$(9, 45)$

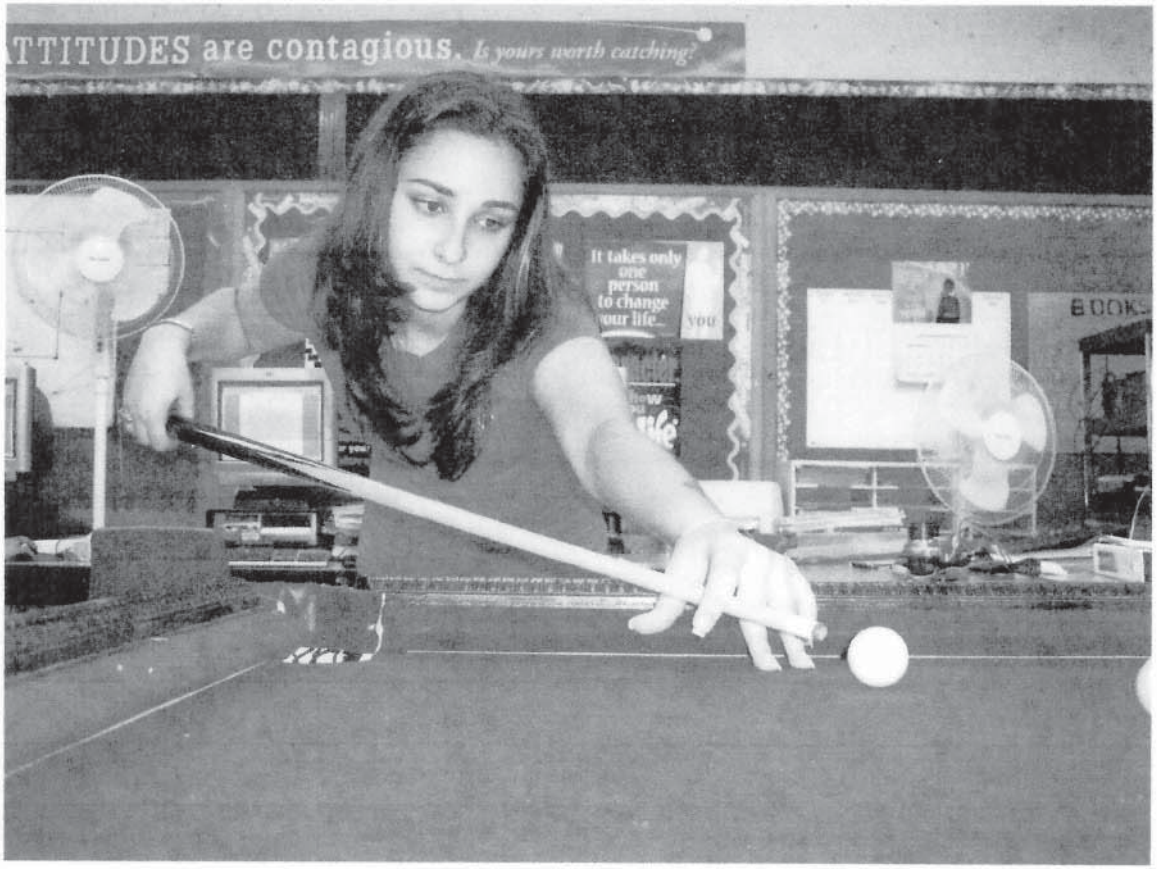
$9 + 1.59 = 10.59$ inches

$45 - 1.59 = 43.41$ inches

Inches (10.59, 43.41)

Feet (.882, 3.62)

Lengths may not be to scale



Ashley's Angle Shot

Ashley's Angle Shot

Angle of Target Ball to Pocket

$\tan^{-1}(1/1.25) = 38.6598$

$\approx \mathbf{38.66^{o}}$

Sides of Smaller Triangle (from point of target to point of second cue ball)

Side a- $\sin 38.66^{o} = a/2.25$

$(2.25)\ .6246976608 = a/2.25\ (2.25)$

1.4 in. = a

Side b- $\cos 38.66^{o} = b/2.25$

$(2.25)\ .7808667188 = b/2.25\ (2.25)$

1.7569

1.76 ≈ b

Point of Target Ball

(8ft., 1.25ft.)

8*12, 1.25*12

(96in, 15in.)

Point of Second Cue Ball

96-1.4 = 94.6

15+1.76 = 16.76

(94.6in., 16.76in.)

/12

(7.883ft., 1.3967ft.)

Point of Contact of Cue Ball and Target Ball

1.4/2 = .7

2.25/2 = 1.125

1.76/2 = .88

96-.7 = 95.3, 15+.88 = 15.88

(95.3in., 15.88in.)

/12

(7.94167ft., 1.323ft.)

Sides of Triangle From Point of Second Cue Ball to Point of First Cue Ball

Side a- 7.883ft.- 6ft. = **1.883ft.**

Side b- 4.5ft.- 1.3967ft. = 3.103ft.

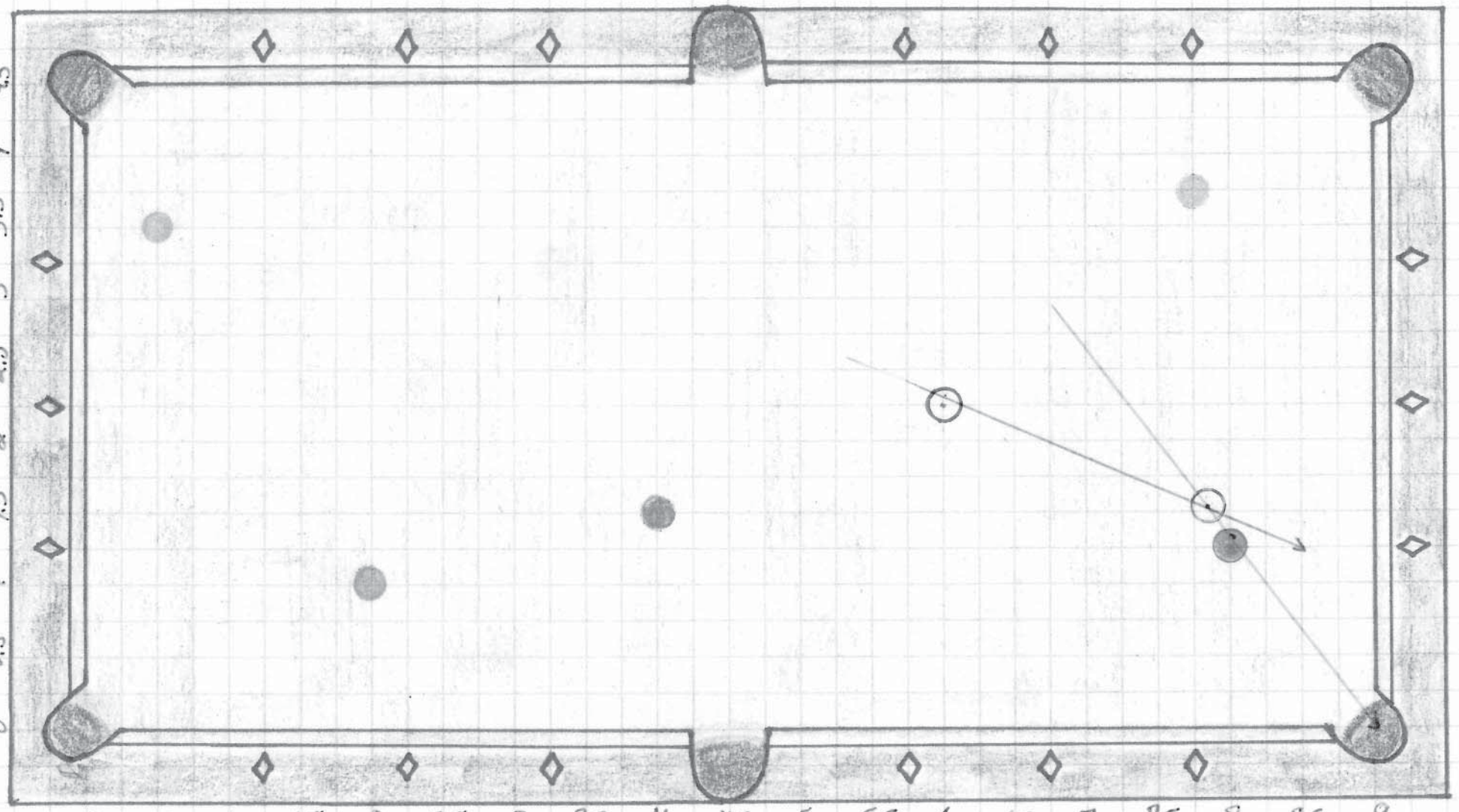
3.103ft.- 2.25ft. = **.853ft.**

Angle Cue Ball Travels In

$\text{Tan}^{-1}(1.883/.853)$

65.624874604

$\approx$ **65.62°**


Width of Table (feet)
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
6
6.5
7
7.5
8
8.5
9
Length of Table (feet)

Angle Target Ball must be hit at

$TAN^{-1}(1/1.25) = 38.6598$

$\approx 38.66°$

Angle Cue Ball Travels In

$7.8\overline{83}$ ft. $- 6$ ft. $= 1.88\overline{3}$ ft.

4.5 ft. $- 1.3967$ ft. $= 3.103$ ft.

3.103 ft. $- 2.25$ ft. $= .85\overline{3}$ ft.

$TAN^{-1}(1.88\overline{3}/.85\overline{3})$

$65.624566.4$

$\approx 65.62°$

Center of Cue Ball at Contact

$\sin 38.66° = \frac{a}{2.25}$

$(2.25).624697608 = \frac{a}{2.25}(2.25)$

1.4 in.

$\cos 38.66° = \frac{b}{2.25}$

$(2.25).780867188 = \frac{b}{2.25}(2.25)$

1.7569

1.76 in. $\approx b$

96 in. $- 1.4$ in. $= 94.6$ in.

15 in. $+ 1.76$ in. $= 16.76$ in.

$(94.6$ in., 16.76 in.$)$

$(7.88\overline{3}$ ft., $1.396\overline{7}$ ft.$)$

Point of Contact Between Cue Ball and Target Ball

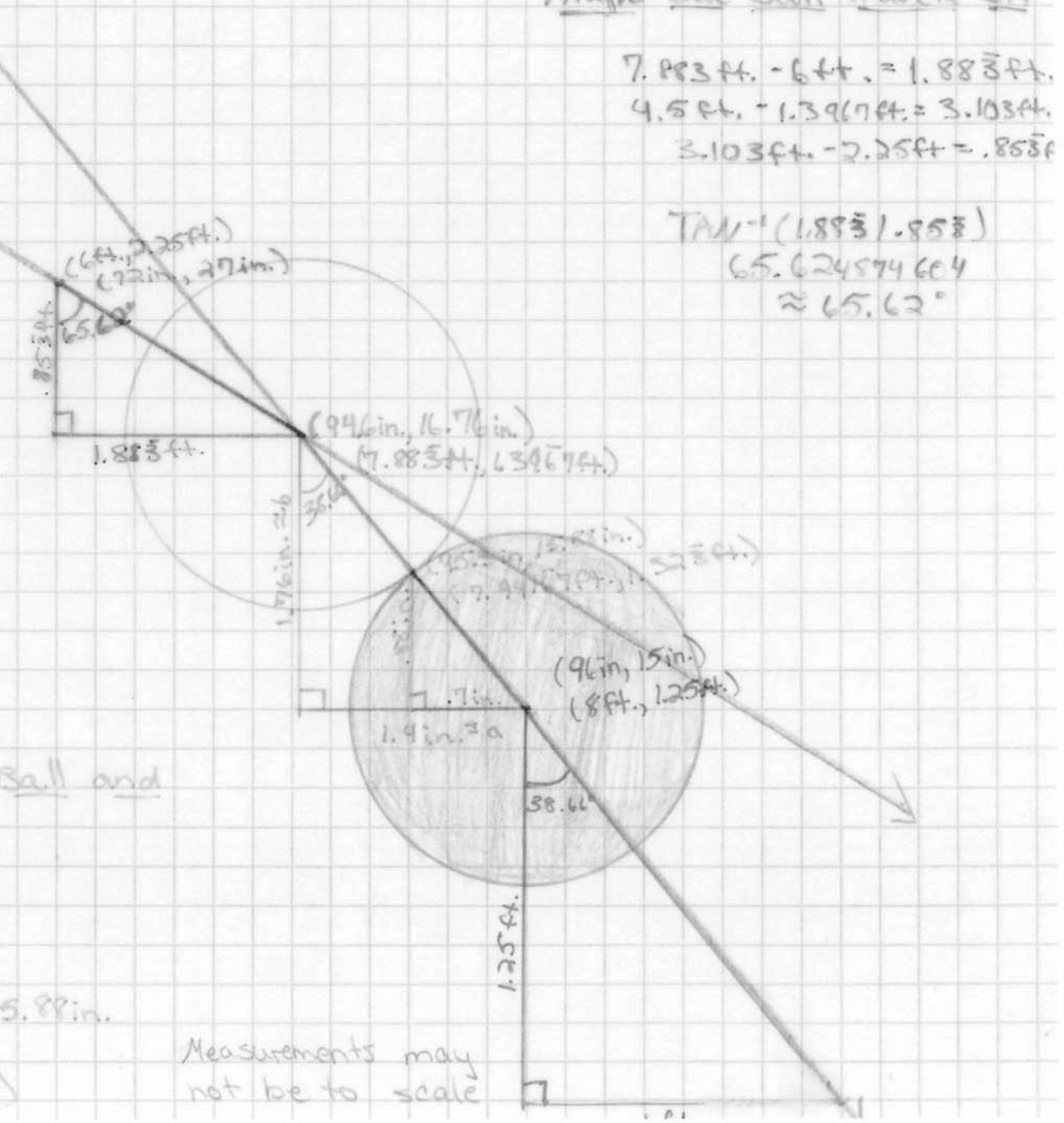
1.4 in. $\div 2 = .7$ in.

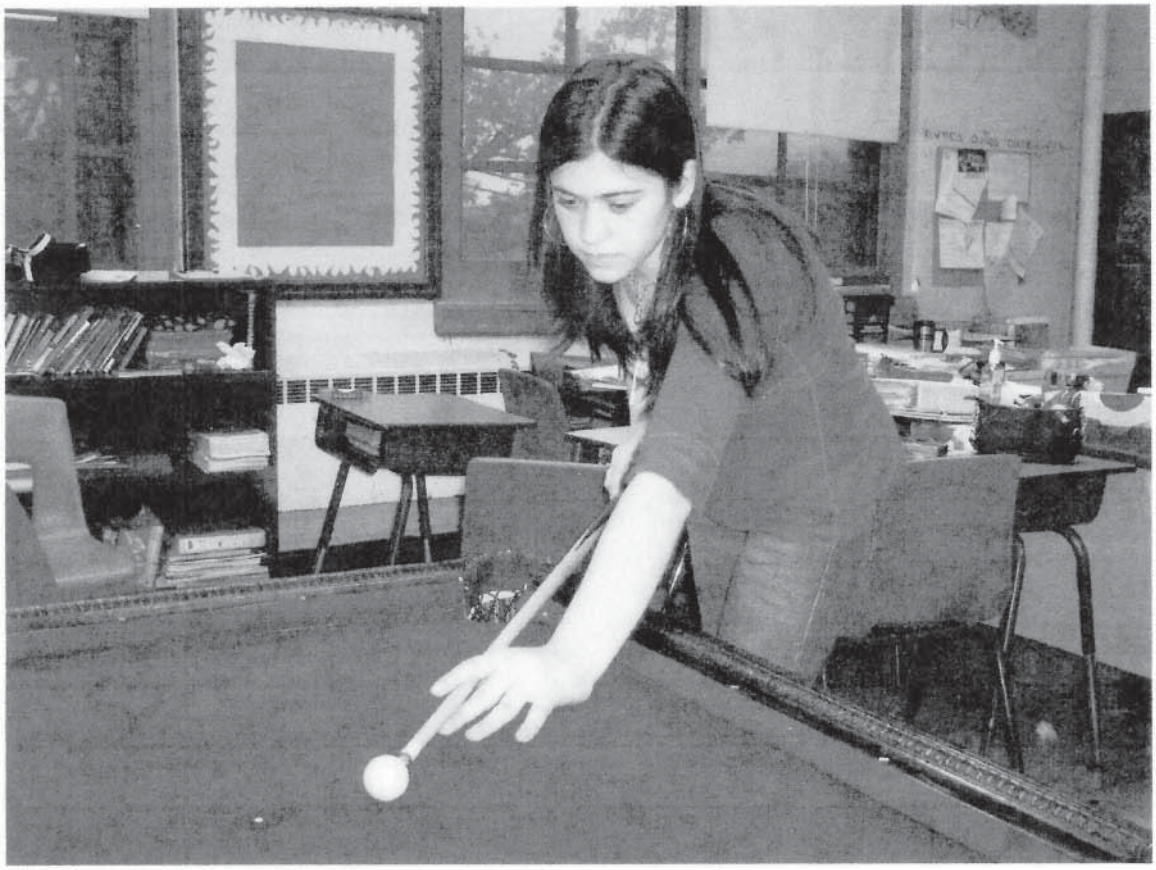
1.76 in. $\div 2 = .88$ in.

96 in. $- .7$ in. $= 95.3$ in., $15 + .88 = 15.88$ in.

$(95.3$ in., 15.88 in.$)$

$(7.941\overline{6}$ ft., $1.32\overline{3}$ ft.$)$





Catherine's Angle Shot

Catherine's Angle Shot

Angle Target ball must be hit at

$\tan^{-1}(1/.75) = 53.13010235$

$\approx \mathbf{53.13^0}$

↑

angle at which target ball is traveling

Center of Cue Ball at Contact

$\text{Cos } 53.13 = a'/2.25$

$(2.25)\ .6000014291 = a'/2.25\ (2.25)$

$a' = 1.350003216$

$\approx \mathbf{1.35}$ **in.**

$\text{Sin } 53.13 = b'/2.25$

$(2.25)\ .7999989281 = b'/2.25\ (2.25)$

$b' = 1.799997588$

$\approx \mathbf{1.8}$ **in.**

(3.75,3.5) ft.

x 12

(45,42) in.

45in.- 1.35 = 43.65

42in.- 1.8 = 40.2

(43.65,40.2) in.

/12

(3.6375, 3.35) ft. → the center point of cue ball

Point of Contact between Cue Ball and Target Ball

1.8/ 2 = .9 in

1.35/ 2 = .675 in

(45,42)

45- .675 = 44.325

42- .9 = 41.1

(44.325, 41.1) in.

/12

(3.69375, 3.425) ft.

↑

Point of Contact- cue ball and target ball meet

Angle Cue Ball is Traveling at

3.35 - 1.5 = 1.85 (x-axis)

3.6375- 3 = .6375 (y-axis)

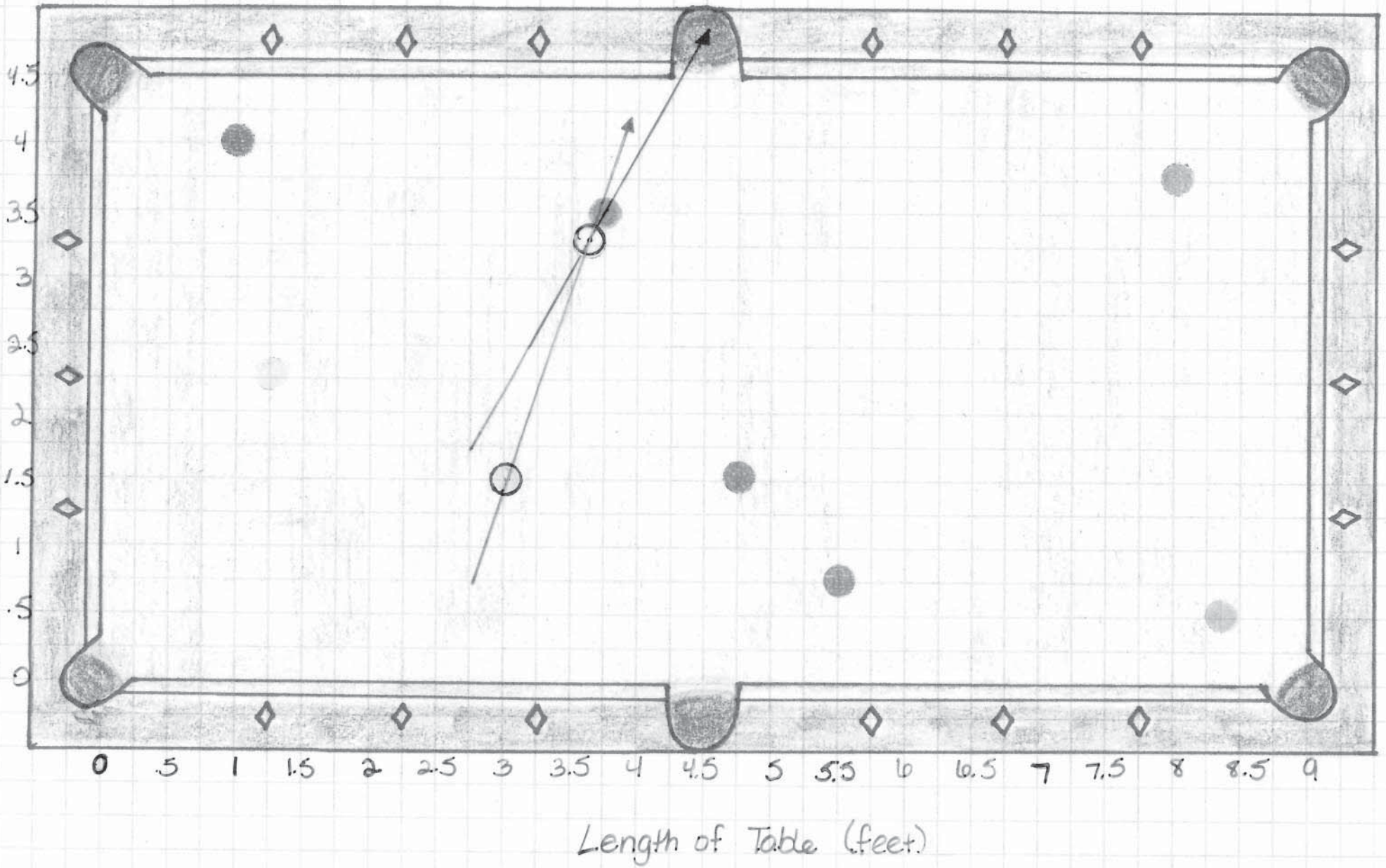
$\tan^{-1}(1.85/.6375)$

$= 70.98632561$

$\approx \mathbf{70.99^{o}}$

↑

angle at which the cue ball is shot


4.5
4
3.5
3
2.5
2
1.5
1
.5
0
0 .5 1 1.5 2 2.5 3 3.5 4 4.5 5 5.5 6 6.5 7 7.5 8 8.5 9
Length of Table (feet)

Angle Target ball must be hit at

$\tan^{-1}\left(\frac{1}{.75}\right) = 53.13010235$
$\approx 53.13°$

Center of Cue Ball at Contact

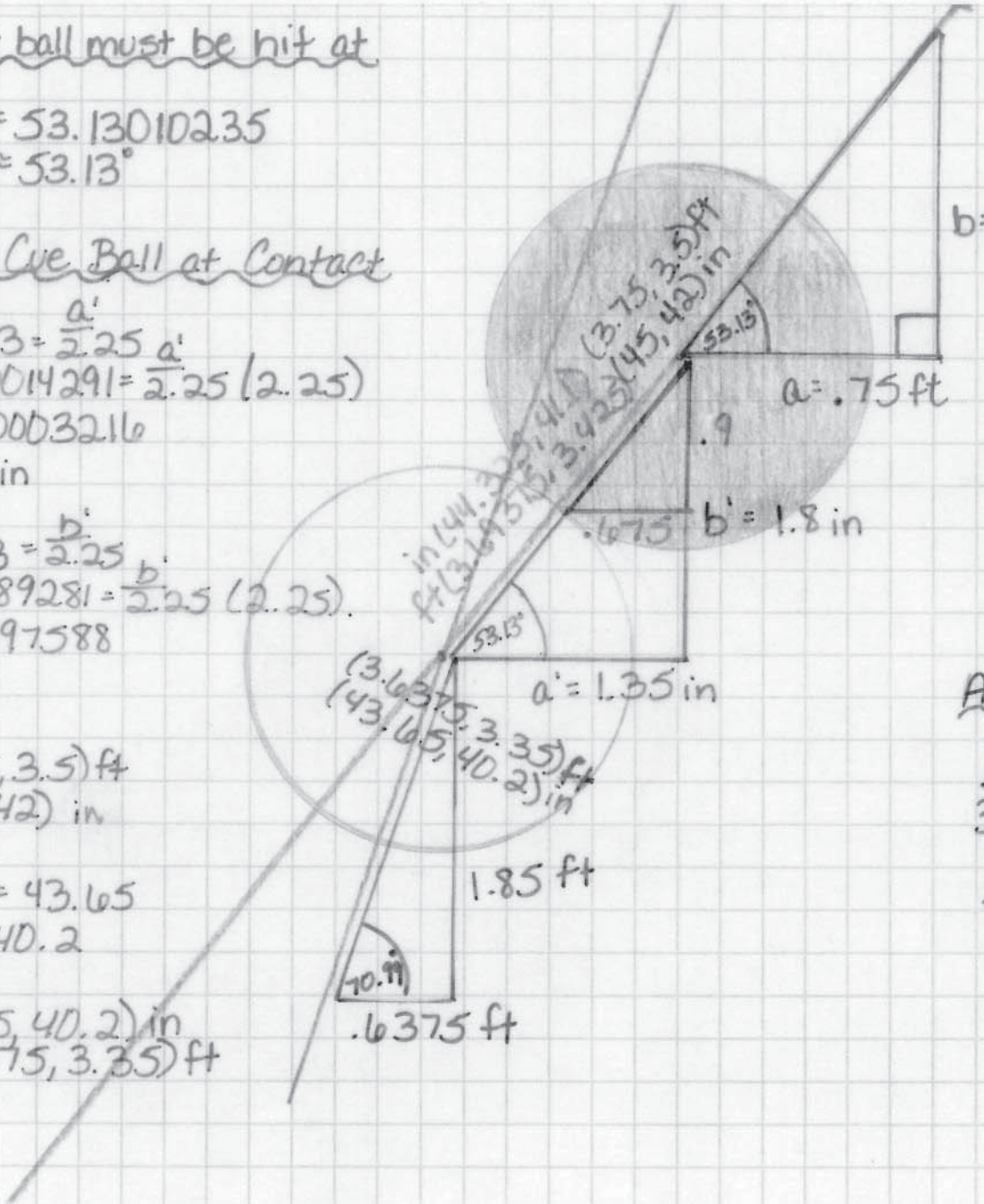
$\cos 53.13 = \frac{a'}{2.25}$
$(2.25).60000014291 = \frac{a'}{2.25}(2.25)$
$a' = 1.3500032116$
≈ 1.35 in

$\sin 53.13 = \frac{b'}{2.25}$
$(2.25).7999989281 = \frac{b'}{2.25}(2.25)$
$a = 1.7999997588$
≈ 1.8 in

(3.75, 3.5) ft
(45, 42) in

45 in - 1.35 = 43.65
42 in - 1.8 = 40.2

(43.65, 40.2) in
(3.6375, 3.35) ft



Point of Contact between cue ball and target ball

(45, 42)
45 - .675 = 44.325
42 - .9 = 41.1

(44.325, 41.1) in
(3.69375, 3.425) ft

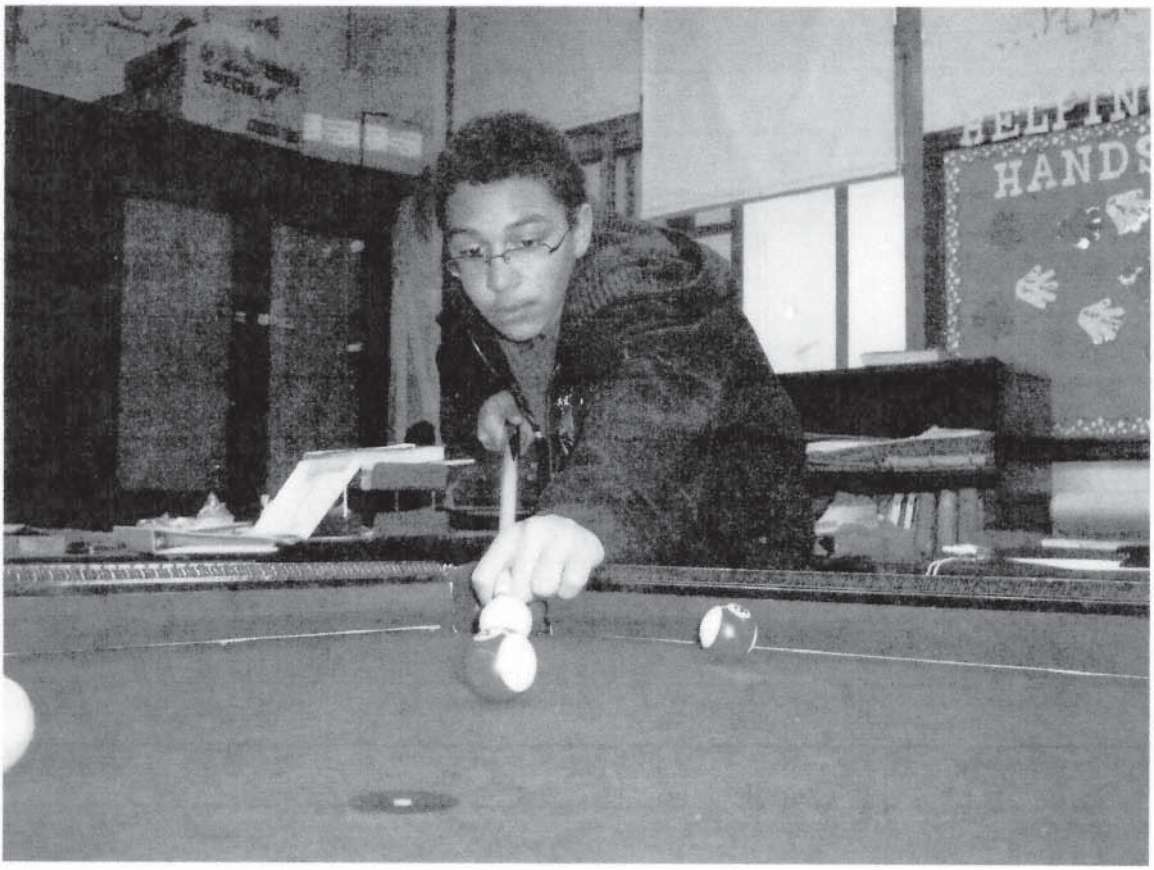
1.8 ÷ 2 = .9
1.35 ÷ 2 = .675

Angle Cue Ball is Traveling at

3.35 - 1.5 = 1.85
3.6375 - 3 = .6375

$\tan^{-1}\left(\frac{1.85}{.6375}\right) = 70.98632561$
$\approx 70.99°$

Measurments may not be to scale



Darius's Angle Shot

Darius's Angle Shot

Angle Target Ball Must Travel At

$TAN^{-1}(1.5/0.5) = 71.5650118^{o} = 71.57^{o}$

$90^{o} + 71.57^{o} = 161.57^{o}$
$180^{o} - 161.57^{o} = 18.43^{o}$

(0.5 ft, 3 ft)

Center of Cue Ball

$SIN(18.4) = \frac{x}{2.25\ in}$

$.3156490369 = \frac{x}{2.25\ in}$

$x = .7102103331\ in = .71\ in = .06\ ft$

$COS(18.4) = \frac{x}{2.25\ in}$

$.9488760116 = \frac{x}{2.25\ in}$

$x = 2.134971026\ in = 2.13\ in = .18\ ft$

$3\ ft - 0.18\ ft = 2.72\ ft * 12 = 32.64\ in$
$0.5\ ft + .06\ ft = .56\ ft * 12 = 6.72\ in$

(.56 ft, 2.72 ft)
(6.72 in, 32.64 in)

Point of Contact Between Cue Ball and Target Ball

$.06\ ft/2 = .03\ ft$
$.18\ ft/2 = .09\ ft$

$0.5\ ft + .03\ ft = .53\ ft * 12 = 6.36\ in$

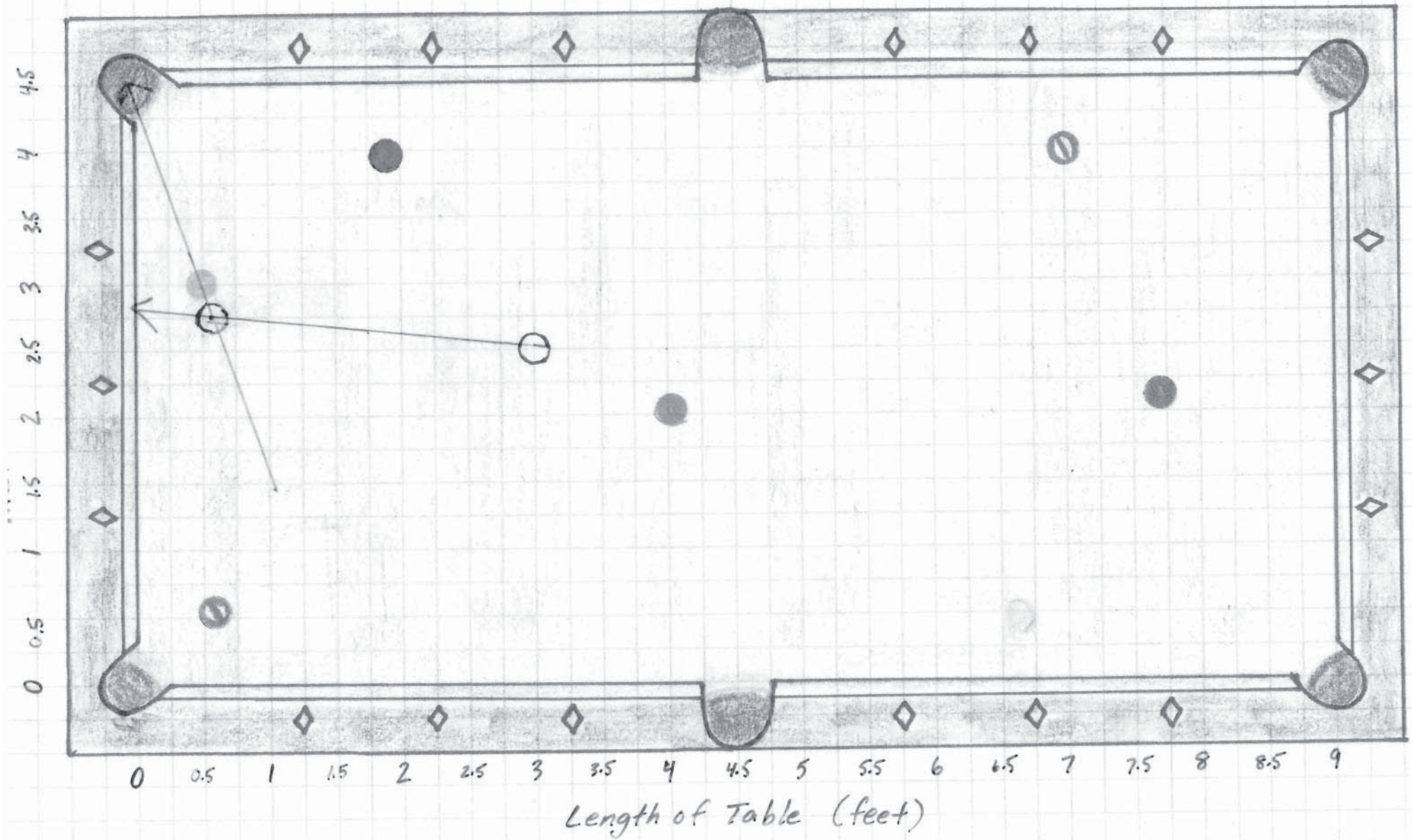
$3\ ft - .09\ ft = 2.91\ ft * 12 = 34.92\ in$

$(.53\ ft, 2.91\ ft)$
$(6.36\ in, 34.92\ in)$

Angle Cue Ball Travels In

$39\ in - 6.72\ in = 32.28\ in$
$32.64\ in - 30\ in = 2.64\ in$

$TAN^{-1}(32.28\ in/2.64\ in) = 85.32450559^{o} = 85.32^{o}$
$90^{o} - 85.32^{o} = 4.68^{o}$


0 0.5 1 1.5 2 2.5 3 3.5 4 4.5
0 0.5 1 1.5 2 2.5 3 3.5 4 4.5 5 5.5 6 6.5 7 7.5 8 8.5 9
Length of Table (feet)

Angle Targetball must be hit at

$TAN^{-1}(1.5/0.5) = 71.56501118°$

$\approx 71.57°$

Center of Cue Ball at Contact

$\sin(18.4) = \frac{x}{2.25in}$

$.3156490369 = \frac{x}{2.25in}$

$x = .7102103331in \approx .71in$

$\approx .06ft$

$\cos(18.4) = \frac{x}{2.25in}$

$.9488760116 = \frac{x}{2.25in}$

$2.134971026in \approx 2.13in$

$\approx .18ft$

$3ft - 0.18ft = 2.72ft * 12 = 32.64in$

$0.5ft + 0.06ft = .56ft * 12 = 6.72in$

$(.56ft, 2.72ft)$

$(6.72in, 32.64in)$

Point of Contact Between Cue Ball and Target Ball

$.06 \div 2 = .03ft$

$0.5ft + 0.03f = .53ft * 12 = 6.36in$

$.18 \div 2 = .09ft$

$3ft - .09ft = 2.91ft * 12 = 34.92in$

$(.53ft, 2.91ft)$

$(6.36in, 34.92in)$

Angle Cue Ball Travels In

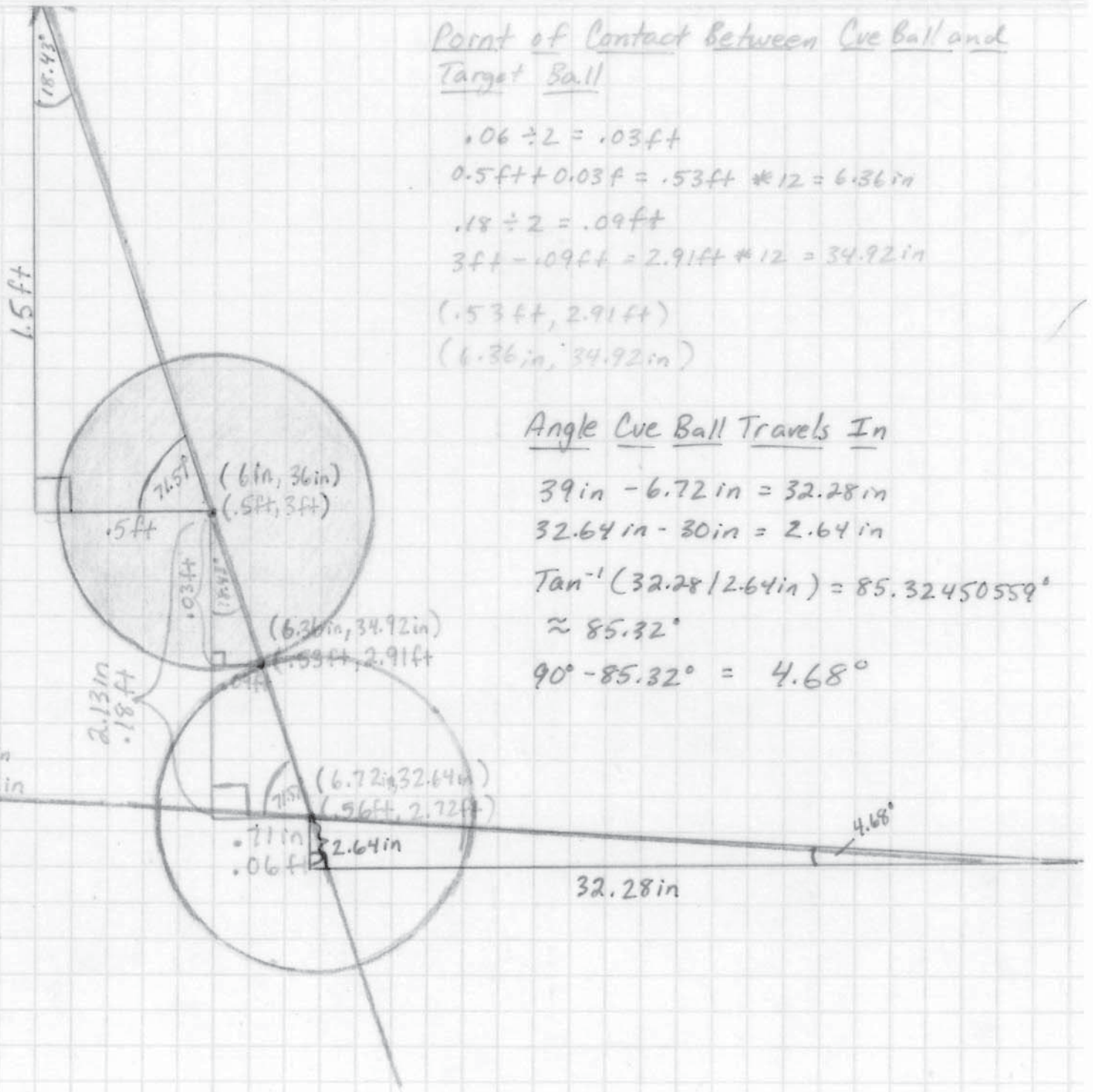
$39in - 6.72in = 32.28in$

$32.64in - 30in = 2.64in$

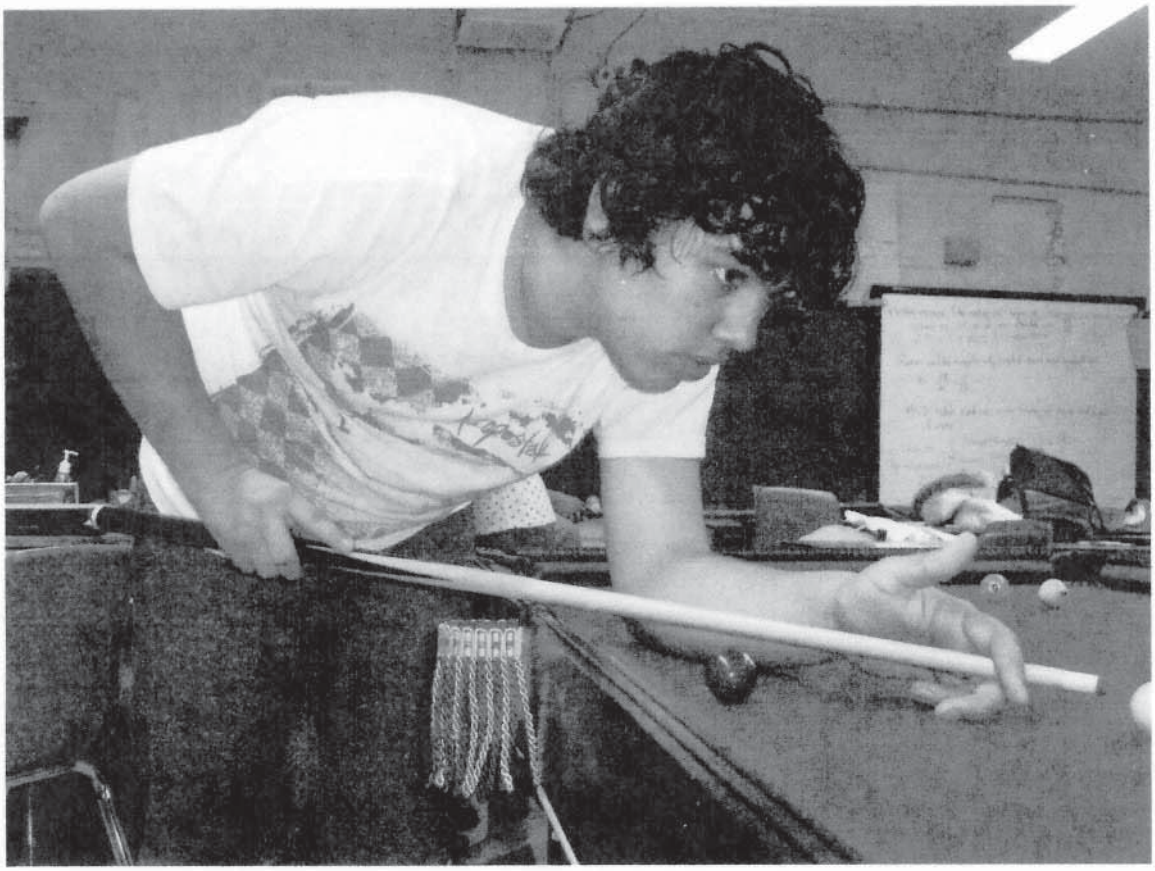
$Tan^{-1}(32.28/2.64in) = 85.32450559°$

$\approx 85.32°$

$90° - 85.32° = 4.68°$



Measurements may NOT BE TO SCALE



John Paul's Angle Shot

John Paul's Angle Shot

Angle Target Ball Must Travel At

$Tan^{-1}=1.5/1$
$56.30993247^{\circ}=$
56.3°

Center Of Cue Ball At Contact

Side of a'

$Cos\ 56.3=a/2.25$
$(2.25)\ .5548444274=a/2.25(2.25)$
$a=1.248399962=$
$a=1.25$ inches

Side of b'

$Sin\ 56.3=b/2.25=$
$(2.25)\ .8319541221=b/2.25\ (2.25)$
$1.871896775=b$
$b=1.87$ inches

Center Of Cue Ball At Contact

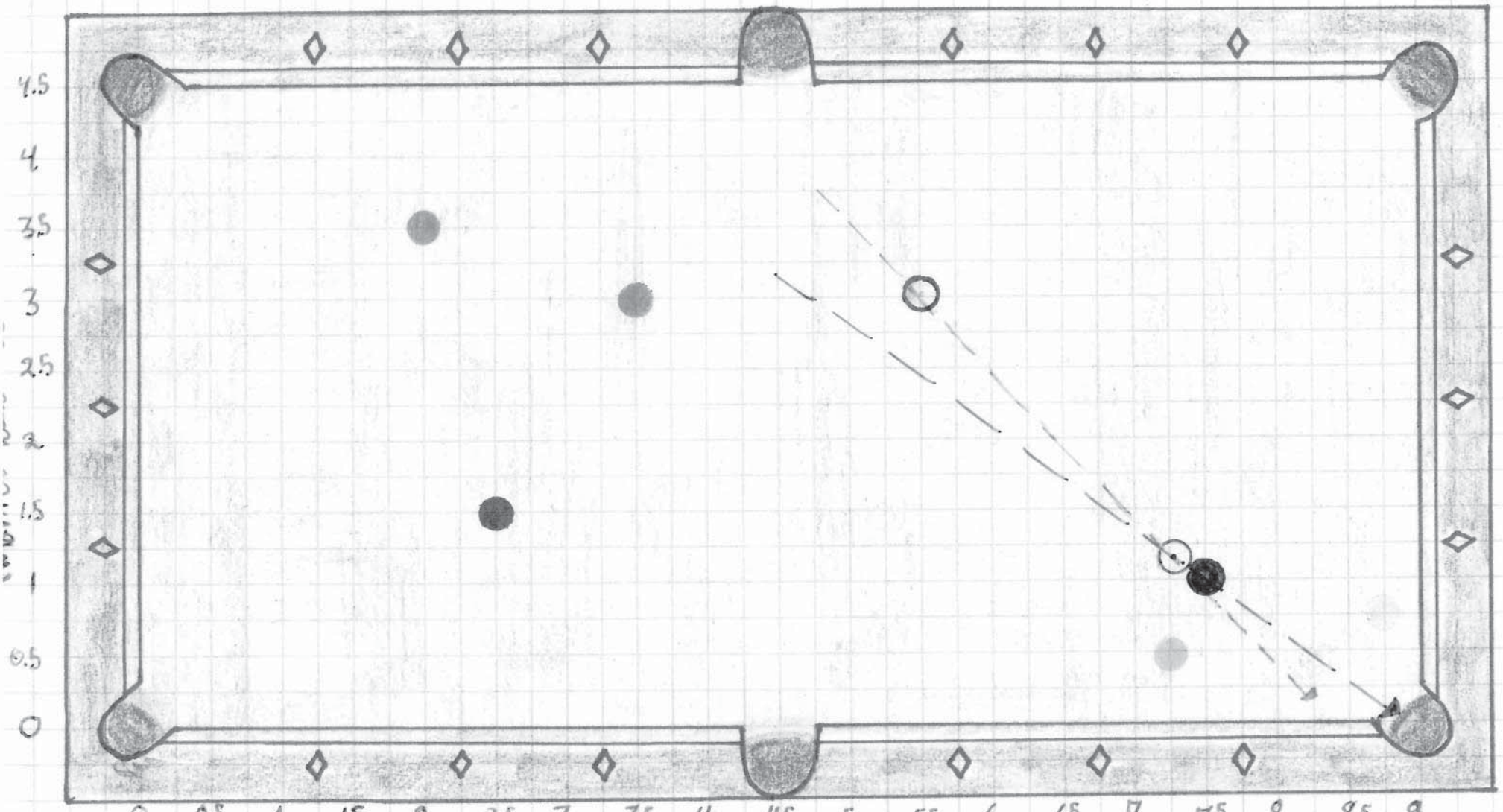
(7 ½, 1) ft
7 ½ *12=90 in 1*12= 12in=
(90, 12) in
90-1.87= 88.13in
12+1.25= 13.25in
(88.13, 13.25) in=
88.13/12=7.34ft
13.25/12=1.11ft
(7.34, 1.11) ft

Point Of Contact

1.87/2= .935in
1.25/2= .625 in
90in -.935=89.065÷12=7.422083333=7.4ft
12in +.625=12.625÷12=1.052083333=1.05ft
(89.065, 12.625) in
(7.4, 1.05) ft

Angle At Which Cue Ball Is Shot

36-13.25= 22.75in
88.13-66=22.13in
Tan^{-1} 22.13/22.75= 44.20853111=
44.21°


4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
0 0.5 1 1.5 2 2.5 3 3.5 4 4.5 5 5.5 6 6.5 7 7.5 8 8.5 9
Length of Table (feet)

Angle Target ball must be hit at

$\tan^{-1} = 1.5/1$

$56.30993247° \approx$

$56.3°$

Center of Cue Ball at Contact

$\cos 56.3 = a/2.25$

$(2.25).55484424 = a/2.25(2.25)$

$a' = 1.248399962 =$

$a' = 1.25$ inches

$\sin 56.3 = b/2.25$

$(2.25).831954122 = b/2.25(2.25)$

$1.871896775 = b'$

$b' = 1.87$ inches

$(7\frac{1}{2}, 1)$ ft.

$(90, 12)$ in.

$90 - 1.87 = 88.13$

$12 + 1.25 = 13.25$

$(88.13, 13.25)$ in $\div 12 =$

$(7.34, 1.11)$ ft.

Point of Contact

$1.25 \div 2 = 1.25$

$1.87 \div 2 = 1.87$

$90 - .935 = 89.065 =$ $(89.065,$

$12 + .625 = 12.625 =$ $12.625)$ in.

$89.065 \div 12 = 7.422083333 = 7.4$

$12.625 \div 12 = 1.052083333 = 1.05$

$(7.4, 1.05)$ ft.

Angle Cue Ball Travels AT

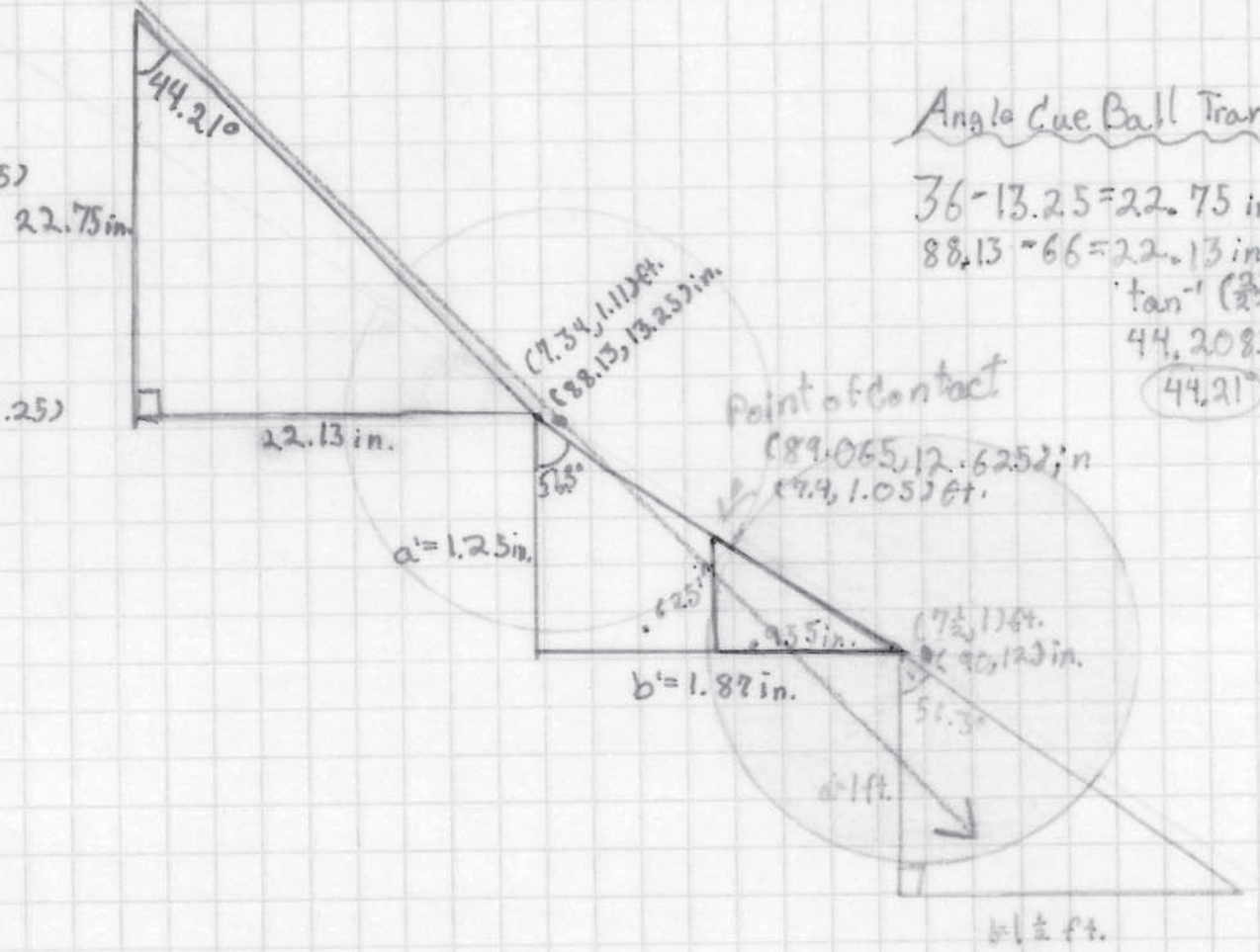
$36 - 13.25 = 22.75$ in.

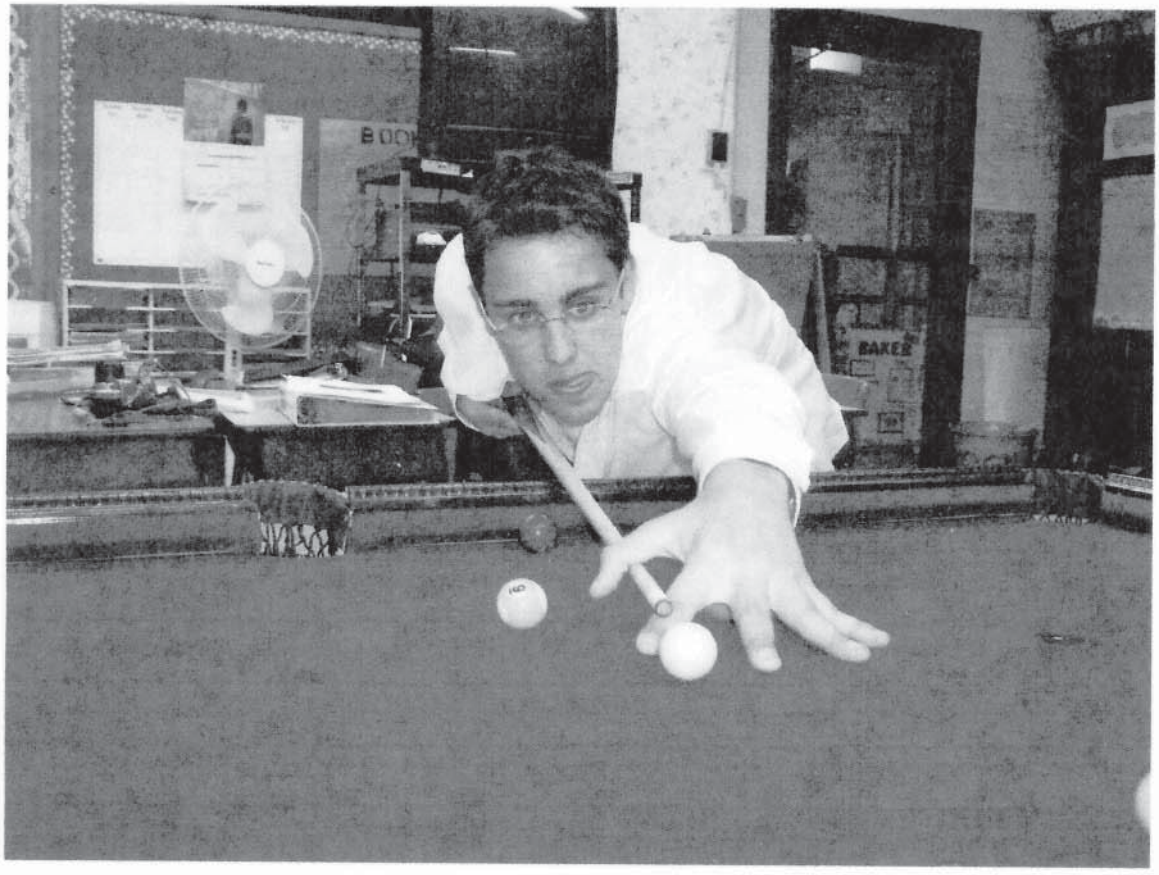
$88.13 - 66 = 22.13$ in.

$\tan^{-1}\left(\frac{22.13}{22.75}\right) =$

$44.20853111° \approx$

$44.21°$





Luis's Angle Shot

Luis's Angle Shot

Angle Target Ball Must be hit at

$\text{Tan}^{-1}(2/1) = 63.43494882° \approx \mathbf{63.43°}$

Center of Cue Ball at Contact

Measure of Side *a*

$\text{Sin } 63.43494882° = {}^{a}/_{2.25}$ in

$(2.25)\ .894427191 = {}^{a}/_{\cancel{2.25}(2.25)}$

$a = 2.01246118$ inches ≈ 2.01 inches

$2.01246118 \div 12 = .1677050983$

$a = .1677050983$ feet $\approx \mathbf{.168\ feet}$

Measure of Side *b*

$\text{Cos } 63.43494882° = {}^{b}/_{2.25}$

$(2.25)\ .4472135955 = {}^{b}/_{\cancel{2.25}(2.25)}$

$b = 1.00623059$ inches ≈ 1.01 inches

$1.00623059 \div 12 = .0838525492$ feet

$b = .0838525492$ feet $\approx \mathbf{.084\ feet}$

Coordinate of Cue Ball at Contact with Target

$(1, 2.5)\ 1 + .0838525492 = 1.0838525492$ feet $\approx \mathbf{1.084\ feet}$

$2.5 - .1677050983 = 2.332294902$ feet $\approx \mathbf{2.33\ feet}$

(1.0838525492 feet, 2.332294902 feet)

$1.0838525492 \times 12 = 13.0062299$ inches $\approx \mathbf{13\ inches}$

$2.332294902 \times 12 = 27.98753885$ inches $\approx \mathbf{28\ inches}$

(13.0062299 inches, 27.98753885 inches)

Coordinate of Contact between Cue and Target

$.0838525492 \div 2 = .0419262746$ feet $\approx \mathbf{.042\ feet}$

$.1677050983 \div 2 = .0838525492$ feet $\approx \mathbf{.084\ feet}$

$1 + .0419262746 = 1.0419262746$ feet $\approx \mathbf{1.04\ feet}$

$2.5 - .0838525492 = 2.416147451$ feet $\approx \mathbf{2.42\ feet}$

(1.0419262746 feet, 2.416147451 feet)

$1.0419262746 \times 12 = 12.5031153$ inches $\approx \mathbf{12.5\ inches}$

$2.416147451 \times 12 = 28.99376941$ inches $\approx \mathbf{28.99\ inches}$

(12.5031153 inches, 28.99376941 inches)

Angle Cue Ball Travels at

Measure of Side *c*

$2.75 - 1.0838525492 = 1.666147451$ feet $\approx$ **1.67 feet**

$1.666147451 \times 12 = 19.99376941$ inches $\approx$ **20 inches**

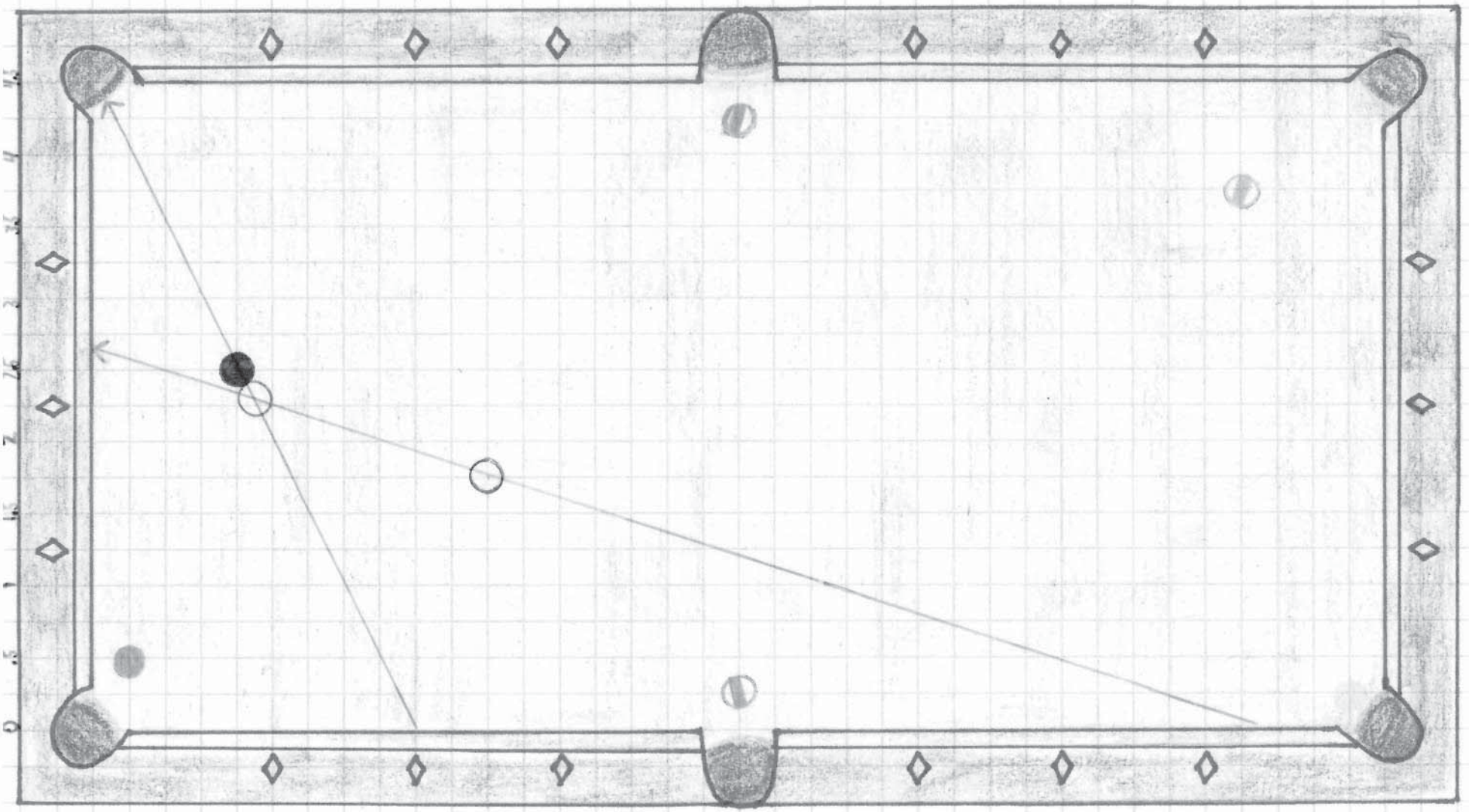
Measure of Side *d*

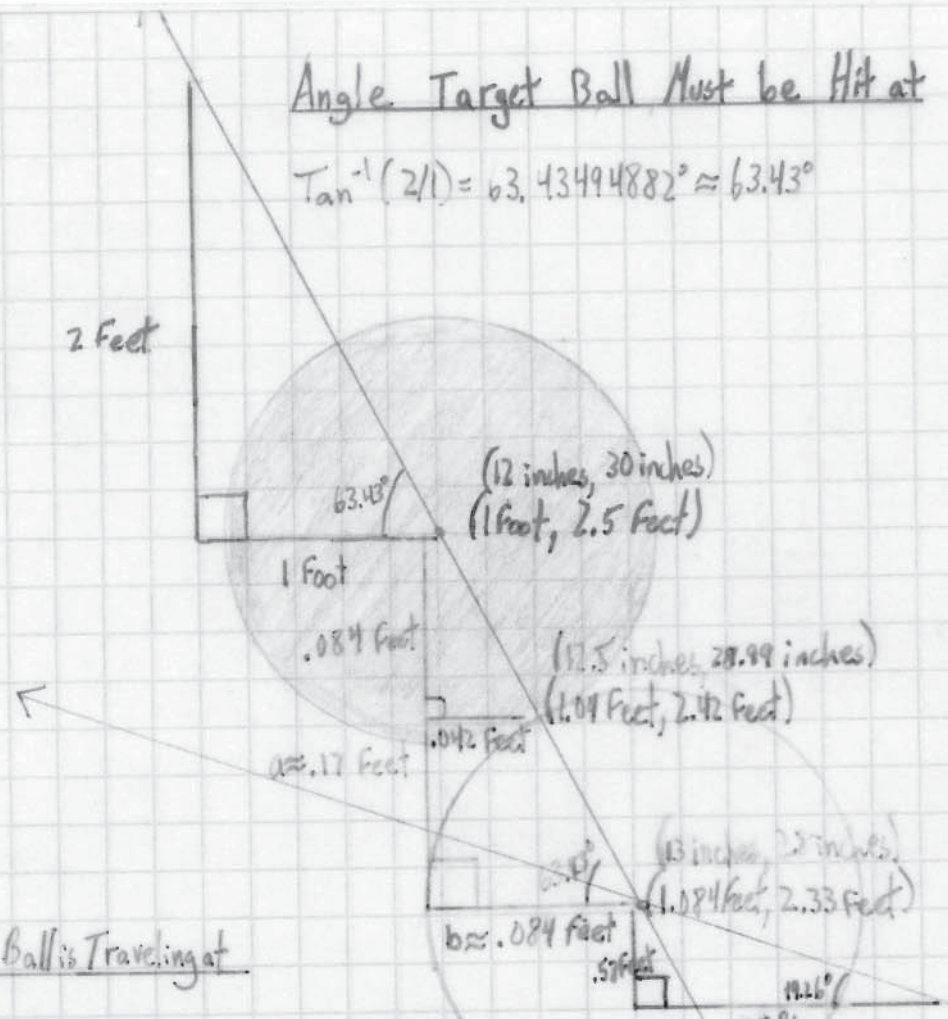
$2.332294902 - 1.75 = .582294906$ feet $\approx$ **.58 inches**

$.582294906 \times 12 = 6.987538872$ inches $\approx$ **6.99 inches**

Angle Cue Ball Travels at

$\text{Tan}^{-1}(^{.582294906}/1.666147451) = 19.26379665° \approx$ **19.26°**


0 .5 1 1.5 2 2.5 3 3.5 4 4.5 5 5.5 6 6.5 7 7.5 8 8.5 9
Length of Table (Feet)



Angle Target Ball Must be Hit at

$\tan^{-1}(2/1) = 63.43494882^\circ \approx 63.43^\circ$

Center of Cue Ball at Contact

$\sin 63.43494882^\circ = a/2.25$ in

$(2.25).89442719 = a/2.25(2.25)$

$a = 2.012460118$ inches ≈ 2.01 inches

$2.012460118 \div 12 = .167705098$

$a = .167705098$ feet $\approx .17$ Feet

$\cos 63.43494882^\circ = b/2.25$ in

$(2.25).44721359 = b/2.25(2.25)$

$b = 1.006230590$ inches ≈ 1.01 inches

$1.006230590 \div 12 = .083852549$ feet

$b = .083852549$ feet $\approx .084$ Feet

$1 + .083852549 = 1.083852549 \approx 1.084$ Feet

$2.5 - .167705098 = 2.332294902$ feet ≈ 2.33 feet

(1.083852549 Feet, 2.332294902 Feet)

$1.083852549 \times 12 = 13.006229900$ inches ≈ 13 inches

$2.332294902 \times 12 = 27.987538850$ inches ≈ 28 inches

(13.006229900 inches, 27.987538850 inches)

Angle Cue Ball is Traveling at

$2.75 - 1.083852549 = 1.666147451$ Feet ≈ 1.66 feet

$1.666147451 \times 12 = 19.993769410$ inches ≈ 20 inches

$2.332294902 - 1.75 = .582294906$ feet $\approx .58$ feet

$.582294906 \times 12 = 6.987538872$ inches ≈ 6.99 inches

$\tan^{-1}(.582294906 / 1.666147451) = 19.26379665^\circ \approx 19.26^\circ$

measurements may not be to scale

Point of Contact

$.083852549 \div 2 = .041926274$ feet $\approx .042$ feet

$.167705098 \div 2 = .083852549$ feet $\approx .084$ feet

$1 + .041926274 = 1.041926274$ feet ≈ 1.04 Feet

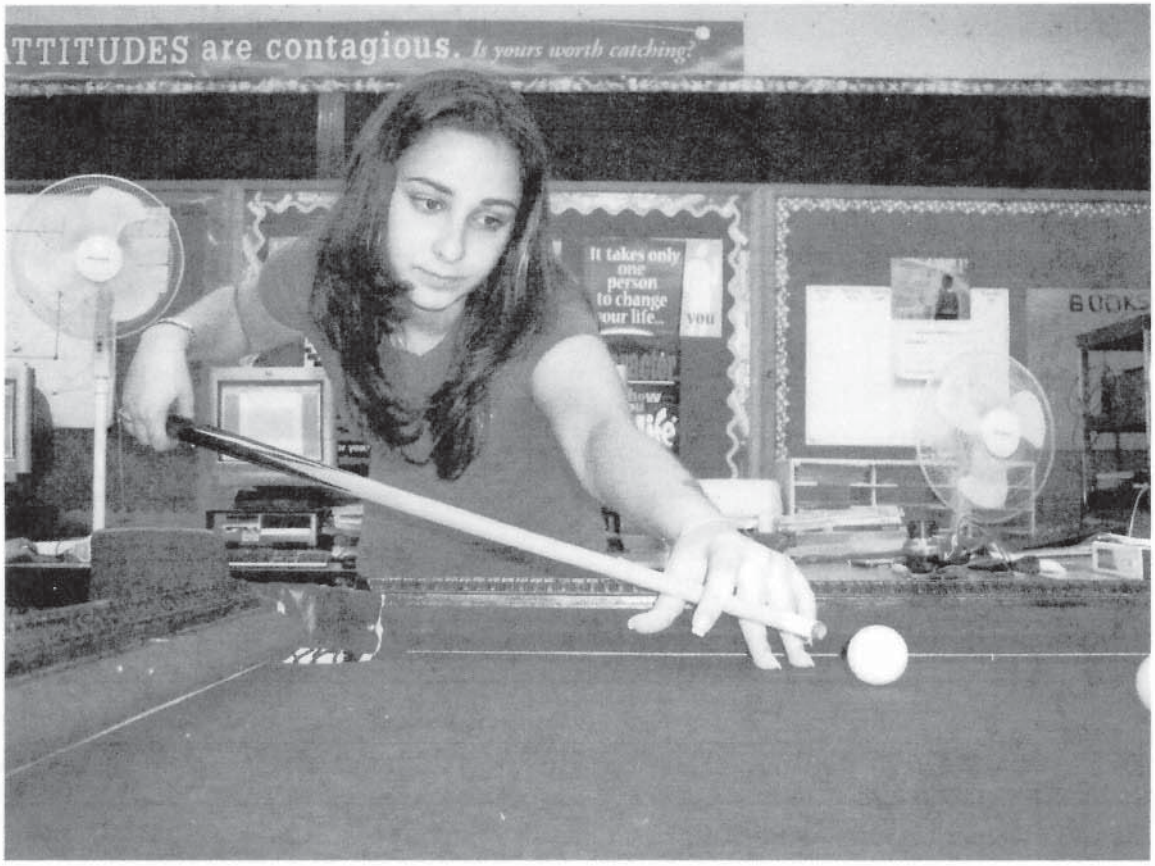
$2.5 - .083852549 = 2.416147451$ feet ≈ 2.42 feet

(1.041926274 feet, 2.416147451 feet)

$1.041926274 \times 12 = 12.503115288$ inches ≈ 12.5 inches

$2.416147451 \times 12 = 28.993769410$ inches ≈ 29 inches

(12.503115288 inches, 28.993769410 inches)



Ashley's Rail Shot

Ashley's Rail Shot

Angle of Target Ball to Pocket

$\tan^{-1}(2/2.25) = 41.63353934$

$\approx \mathbf{41.6^{\circ}}$

Sides of Smaller Triangle (point of target to point of second cue ball)

Side a- $\sin 41.6^{\circ} = a/2.25$

$(2.25)\ .6639262127 = a/2.25\ (2.25)$

1.493833978

$\mathbf{1.49 \approx a}$

Side b- $\cos 41.6^{\circ} = b/2.25$

$(2.25)\ .7477980905 = b/2.25\ (2.25)$

1.682545704

$\mathbf{1.68 \approx b}$

Center of Target Ball

(2ft., 2.25ft.)

$\times 12$

(24in., 27in.)

Center of Cue Ball at Point of Contact

$24+1.49 = 25.49$

$27+1.68 = 28.68$

(25.49in., 28.68in.)

$/12$

(2.124167ft., 2.39ft.)

Point of Contact of Cue Ball and Target Ball

Target- (2ft.,2.25ft.)

(24in.,27in.)

$1.49/2 = .745$in.

$1.68/2 = .84$in.

24in.+.745in. = 24.745in.

27in.+.84in. = 27.84in.

(24.745in.,27.84in.)

$/12$

(2.062083ft.,2.32ft.)

Finding Lengths of Similar Triangles

4.5ft.-1.75ft. = 2.75ft.

2.75ft.×12 = **33in.**-height of triangle outside the table

4.5ft.×12 = 54in.

54in.- 28.68in. = **25.32in.**-height of the rest of triangle in the table

4ft.×12 = 48in.

48in.- 25.49in. = **22.51in**.- base of big triangle

25.32in.+ 33in. = **58.32in.**-height of the big triangle

Proportion to Find Missing Length of the Similar Triangles

$$\frac{33\text{in.}}{58.32\text{in.}} = \frac{x}{22.51\text{in.}}$$

$$\frac{742.83\text{in.}}{58.32\text{in.}} = \frac{58.32\text{in.}x}{58.32\text{in.}}$$

$12.73713992 = x$

$12.74\text{in.} \approx x$

/12

$1.06167\text{ft.} \approx x$

Point of Contact on Rail

48in.- 12.74in. = 35.26in.

35.26in./12 = 2.9383ft.

(2.9383ft.,4.5ft.)

*12

(35.26in., 54in.)

Angle of Reflection

Tan^{-1} (33in./12.74in.)

68.89038959

$\approx 68.89°$

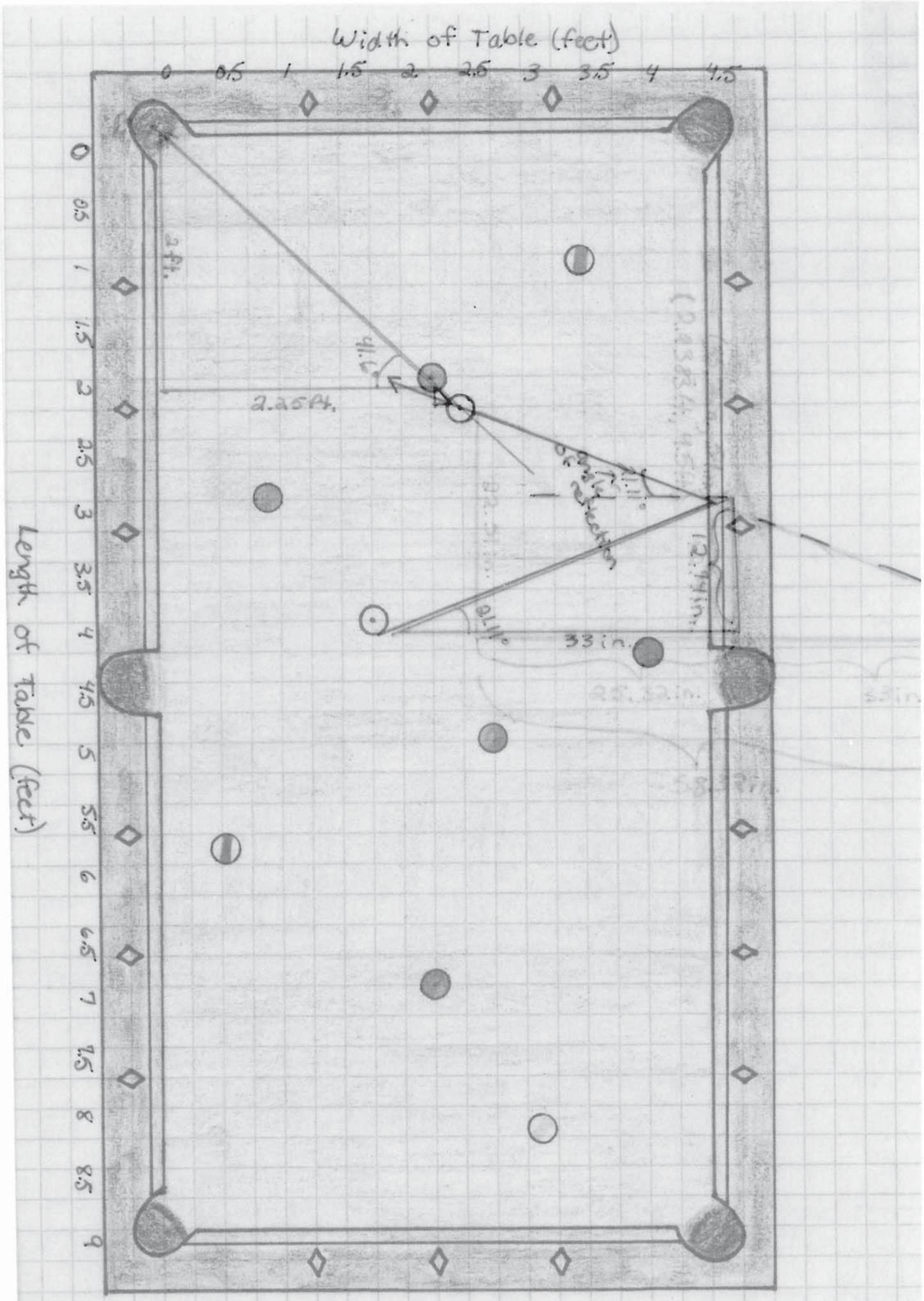
90-68.89 = **$21.11°$**

Angle at Which Cue Ball is Struck

Tan^{-1} (12.74in./33in.)

21.10961041

$\approx 21.11°$


Width of Table (feet)
0 0.5 1 1.5 2 2.5 3 3.5 4 4.5
Length of Table (feet)
0 0.5 1 1.5 2 2.5 3 3.5 4 4.5 5 5.5 6 6.5 7 7.5 8 8.5 9
2 ft.
41.6°
2.25 ft.
11.1°
(2.14 in.)
41.6°
33 in.

Point of Contact on Rail

$\frac{33in.}{58.32in.} = \frac{x}{22.51in.}$

$33 \times 22.51 \div 58.32 = 12.737\ldots$

$12.74in.$

$48in. + 12.74in. = 35.26in.$ + D

$(2.935\ldots, 4.50)$

$(35.26in., 54in.)$

Angle Cue Ball is Struck At

$TAN^{-1}(12.74in./33in.)$

$21.109\ldots$

$\approx 21.11°$

Angle of Reflection

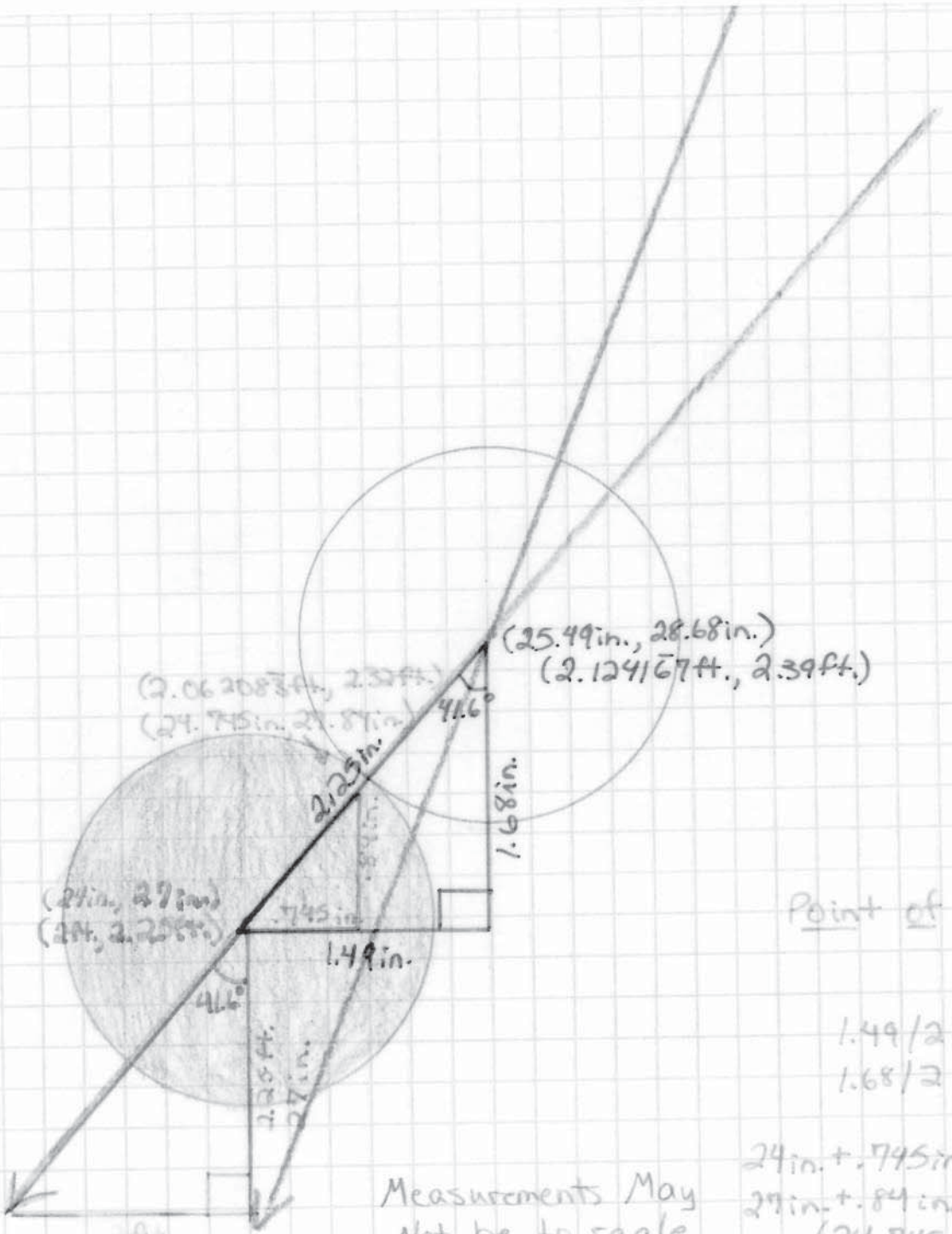
$TAN^{-1}(33in./12.74in.)$

$68.890\ldots$

$\approx 68.89°$

$90° - 68.89° = 21.11°$

* Work in Pink on different transparencies



Angle Target Ball Travels $\angle T$

$\tan^{-1}(2/2.25) = 41.633539...$
$\approx 41.6°$

Center of Cue Ball

$\sin 41.6° = \frac{a}{2.25}$

$(2.25).663926212^{7} = \frac{a}{2.25}(2.25)$

1.493833978

$1.49 \approx a$

$\cos 41.6° = \frac{b}{2.25}$

$(225).7477980905 = \frac{b}{2.25}(2.25)$

1.682545704

$1.68 \approx b$

$24 + 1.49 = 25.49$ in.

$27 + 1.68 = 28.68$ in.

(25.49in., 28.68in.)

(2.124167 ft., 2.39ft.)

Point of Contact Between Cue Ball and Target Ball

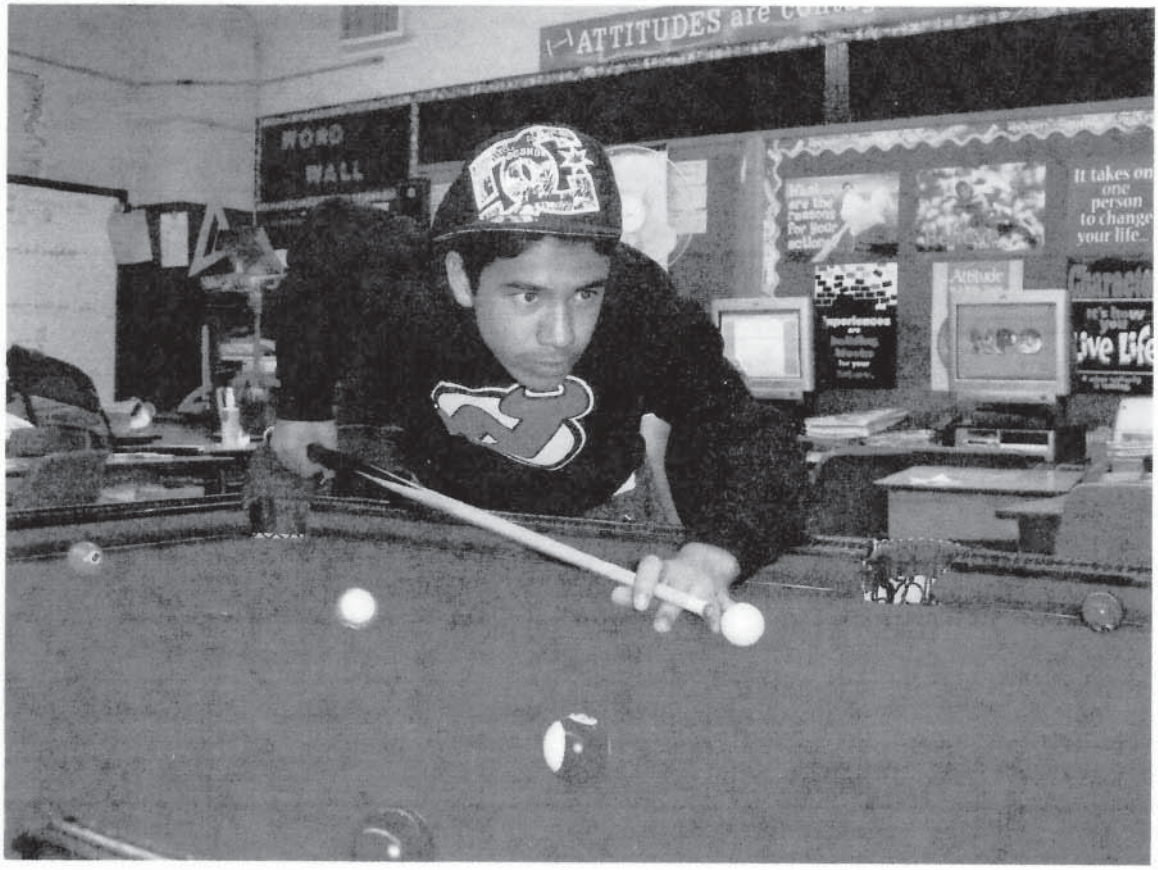
$1.49/2 = .745$ in.

$1.68/2 = .84$ in.

$24\text{in.} + .745\text{in.} = 24.745$ in.

$27\text{in.} + .84\text{in.} = 27.84$ in.

(24.745 in., 27.84in.) (2.062083ft., 2.32ft)



Rodrigo's Rail Shot

Rodrigo's Rail Shot

Angle Target Ball Travels At

$\tan^{-1}(.75/.25)=71.56505118$

$\approx 71.6^{\circ}$

Finding The Center Of The Cue Ball At Contact

Lengths of Smaller Triangle (from the center of the target ball to the cue ball)

Side a-$\sin 71.6^{\circ} = a/2.25$

$(2.25).9488760116 = a/2.25(2.25)$

$2.134971026 = a$

2.14in. ≈ a

Side b-$\cos 71.6^{\circ} = b/2.25$

$(2.25).3156490369 = b/2.25(2.25)$

$.7102103331 = b$

.71 in. ≈ b

Center of Target Ball

(4.25, 3.75)ft.

*12

(51, 45)in.

Center of Cue Ball at Point of Contact

51-.71, 45-2.14

(50.29, 42.86)in.

/12

(4.19083, 3.57167)ft.

Point of Contact Between Cue Ball and Target Ball

Target- (51in.,45in.)

.71/2 = .355in.

2.14/2=1.07in.

51-.355=50.645in.

45- 1.07 = 43.93in.

(50.645in., 43.93in.)

/12

(4.220416667ft., 3.66083333ft.)

Finding the Lengths of Different Triangles

30in.- .71in. =29.29in.- base of big triangle

36in.+42.86in.= 78.86in.- height of big triangle

3.75ft. * 12 = 45in.

45in.- 2.14in. = 42.86in.- height of smaller triangle

Proportion to Find Missing Length of the Similar Triangles

$$\frac{42.86in.}{78.86\ in.} = \frac{x}{29.29in.}$$

$$\frac{1.255.3694in.}{78.86in.} = \frac{78.86in.x}{78.86in.}$$

$$15.91896272 = x$$

$$\mathbf{15.92in. \approx x}$$

Point of Contact on Rail

50.29in. - 15.92in. = 34.37in.

(34.37in.,0in.)

/12

(2.86ft., 0ft.)

Angle of Reflection

$\tan^{-1}$ (42.86in./15.92in.)

69.62289351

$\approx 69.62^{\circ}$

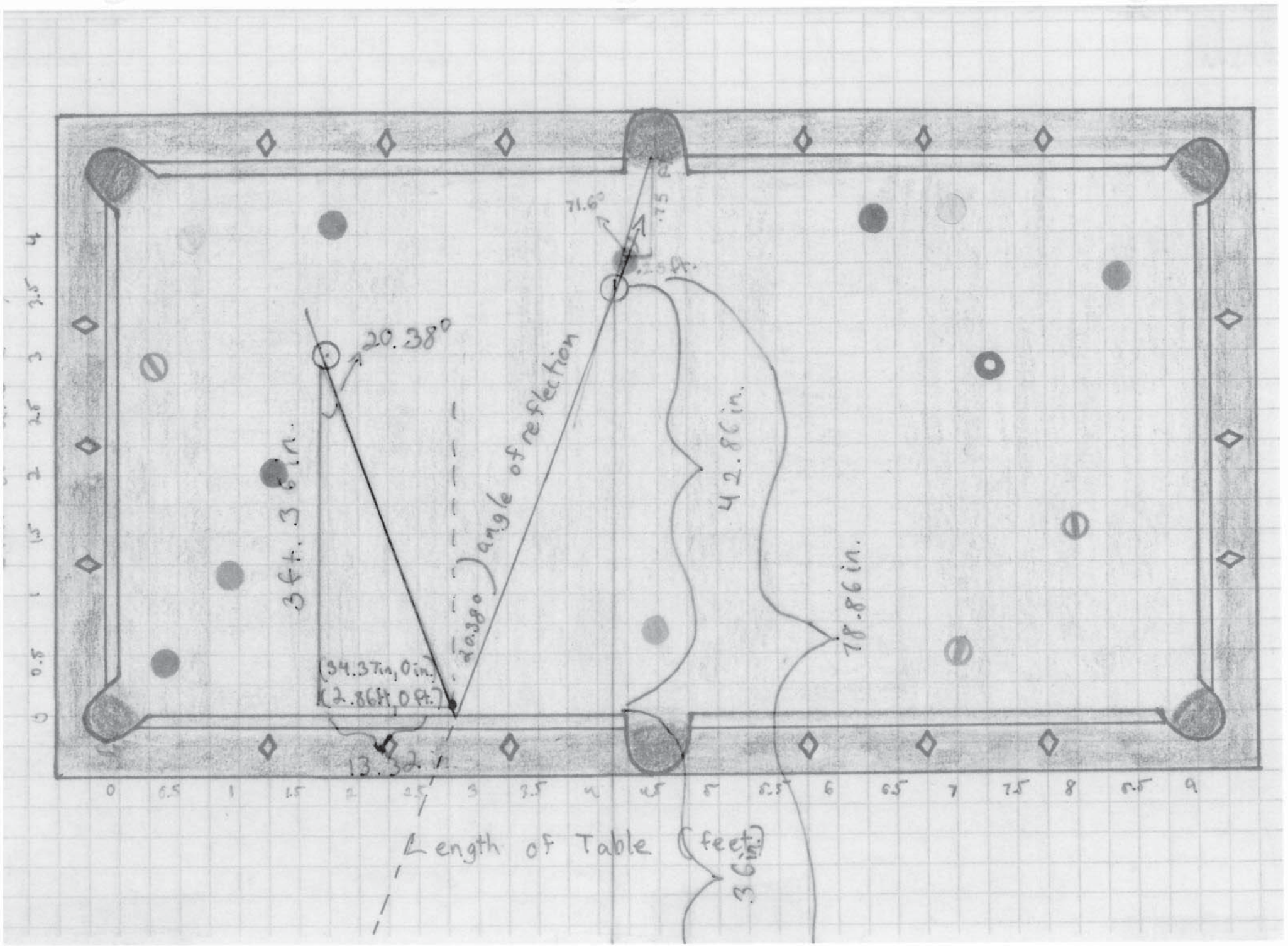
$90-69.62=\mathbf{20.38^{\circ}}$

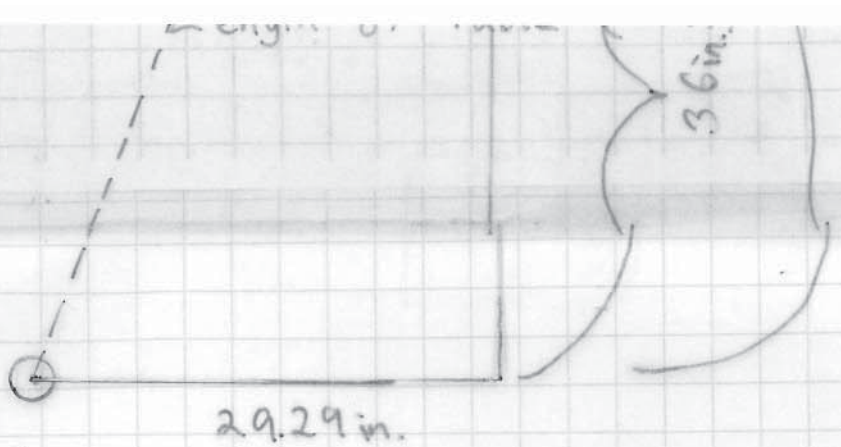
Angle at Which Ball is Struck

$\tan^{-1}$ (15.92in./42.86in.)

20.37710649

$\approx 20.38^{\circ}$


71.6°
20.38°
3ft. 3.5in.
(54.37in, 0in.)
(2.86ft, 0ft.)
13.32in.
(20.38° angle of reflection
42.86in.
18.86in.
Length of Table (feet)
36in.
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
6
6.5
7
7.5
8
8.5
9



Point of Contact on Rail

$$\frac{42.86\text{ in.}}{73.86\text{ in.}} = \frac{x}{29.29\text{ in.}}$$

$29.29 \times 42.86 \div 73.86 = 15.91896272$

$15.92 \approx x$

$50.29\text{ in.} - 15.92\text{ in.} = 34.37\text{ in.} \div 12$

2.86 ft

(34.37 in., 0 in.)

(2.86 ft., 0 ft.)

Angle Cue Ball is Struck At

$\text{Tan}^{-1}(15.92\text{in.}/42.86\text{in.})$

20.37710649

$\approx 20.38°$

Angle of Reflection

$\text{Tan}^{-1}(15.92\text{in.}/42.86\text{in.})$

20.37710649

$\approx 20.38°$

Angle at Which Target Ball is Traveling

$\tan^{-1}(.75 \div .25) = 71.56505118$

$\approx 71.6°$

Center of Cue Ball

Side a - $\sin 71.6° = a/2.25$

$(2.25).9488760116 = a/2.25\,(2.25)$

$2.134971026 = a$

$a \approx 2.14$ in.

Side b - $\cos 71.6° = b/2.25$

$(2.25).315649036 = b/2.25\,(2.25)$

$.7102103371 = b$

$b \approx .71$ in.

51 - .71, 45 - 2.14

(50.29, 42.86) in.

(4.19083, 3.57167) ft.

Point of Contact Between Cue Ball and Target

$.71 \div 2 = .355$ in

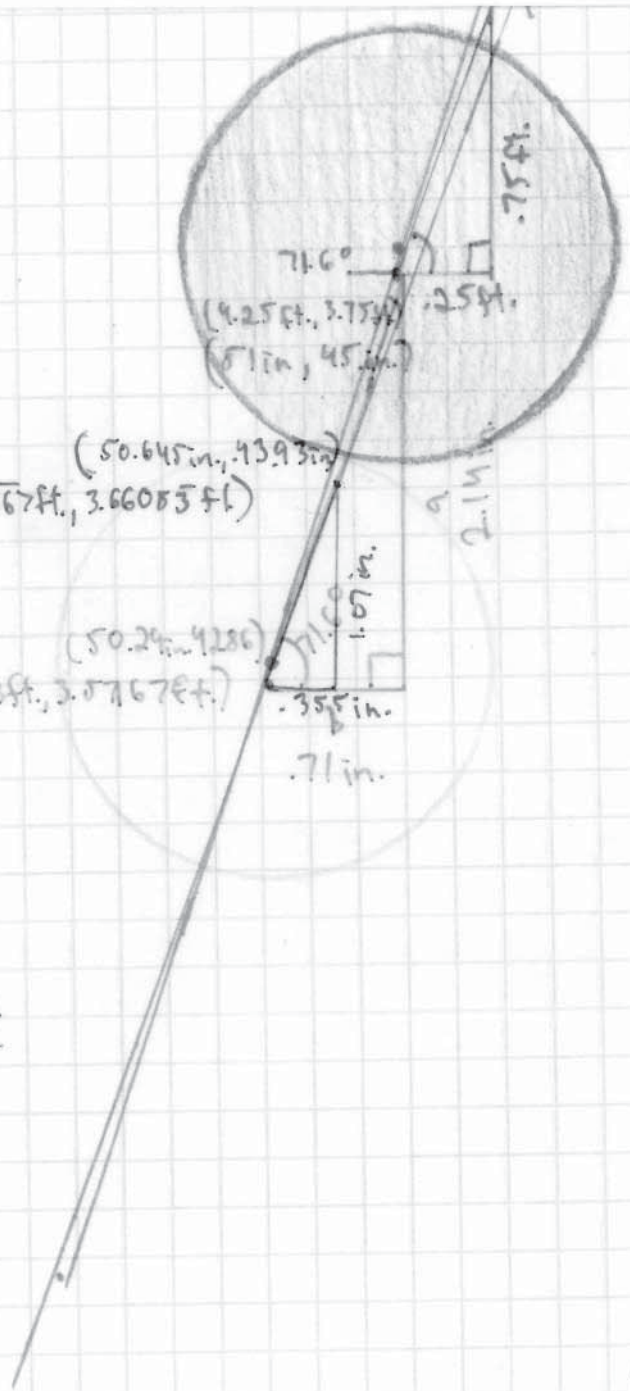
$2.14 \div 2 = 1.07$ in.

$51 - .355 = 50.645$ in.

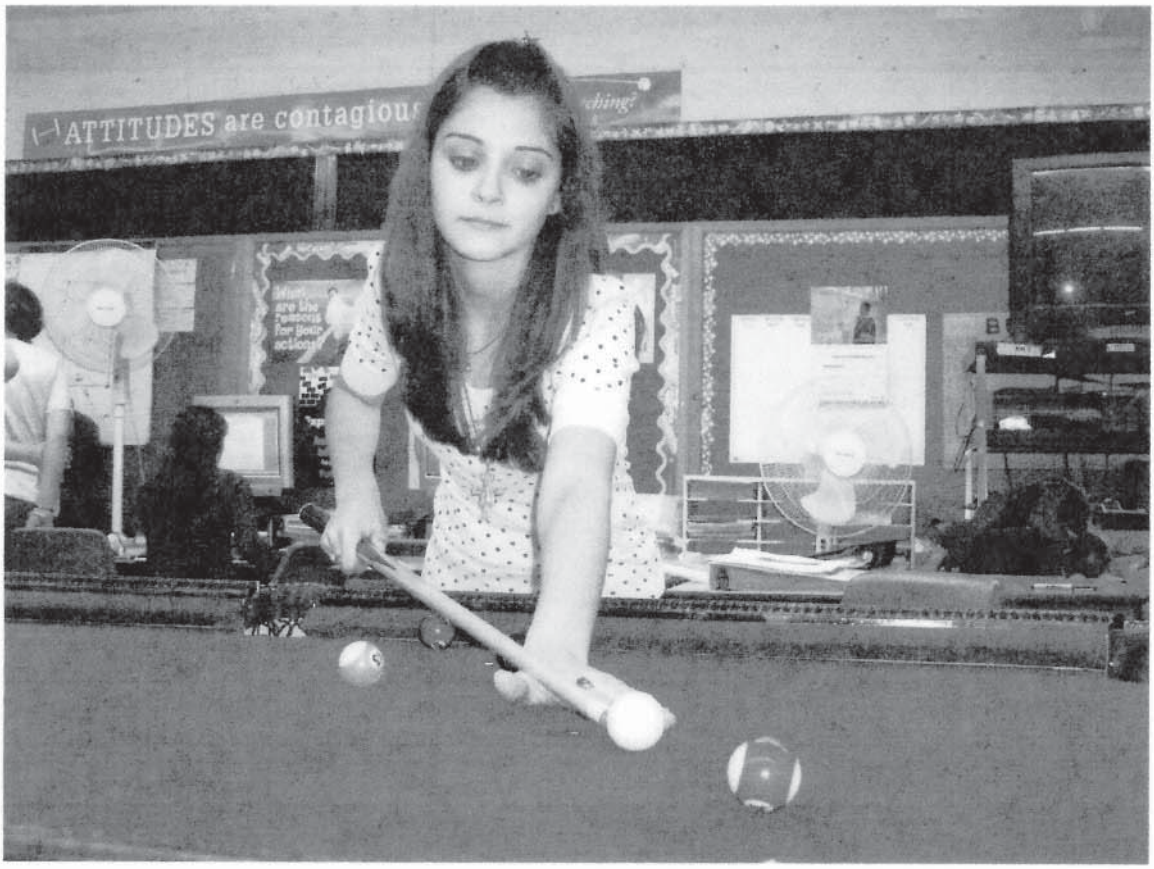
$45 - 1.07 = 43.93$ in.

(50.645 in., 43.93 in.)

(4.220416667 ft., 3.660833333 ft.)



Measurements may Not Be to Scale



Jessica's Rail Shot

Jessica's Rail Shot

Angle At Which Target Ball is Traveling

Tan-1(1.75 ft./3 ft.)= 30.26°

Center of Cue Ball

Sin 30.26° = a/2.25
.5039247371 = a/2.25
1.133830658 = a a = 1.13 in.
Cos30.26° = b/2.25
.8637475669 = b/2.25
1.94342026 = b b= 1.94 in.
Convert **(1.75,1.5 ft.)** to inches
1.75ft.×12 in. = 21 in.
1.5 ft.×12 in. = 18 in.
(21,18 in.)
21 in.+1.13in. = 22.13 in.
18 in.-1.94 in. = 16.06 in.
(22.13,16.06 in.)

Point of Contact Between Cue Ball and Target Ball

2.25 in.÷2 = 1.125 in.
1.13 in.÷2 = .565 in.
1.94 in.÷2 = .97 in.
21 in.+.565 in. = 21.565 in.
18 in.-.97 in. = 17.03 in.
(21.565,17.03 in.)

Point of Contact on Rail

16.06+12 = 28.06 in.
1.25 ft.×12 in. = 15 in.
15 in.-1.13 in. = 13.87 in.
16.06/28.06 = x /13.87
222.7522/28.06 = 28.06/28.06
X = 7.938424804
X = 7.94 in.

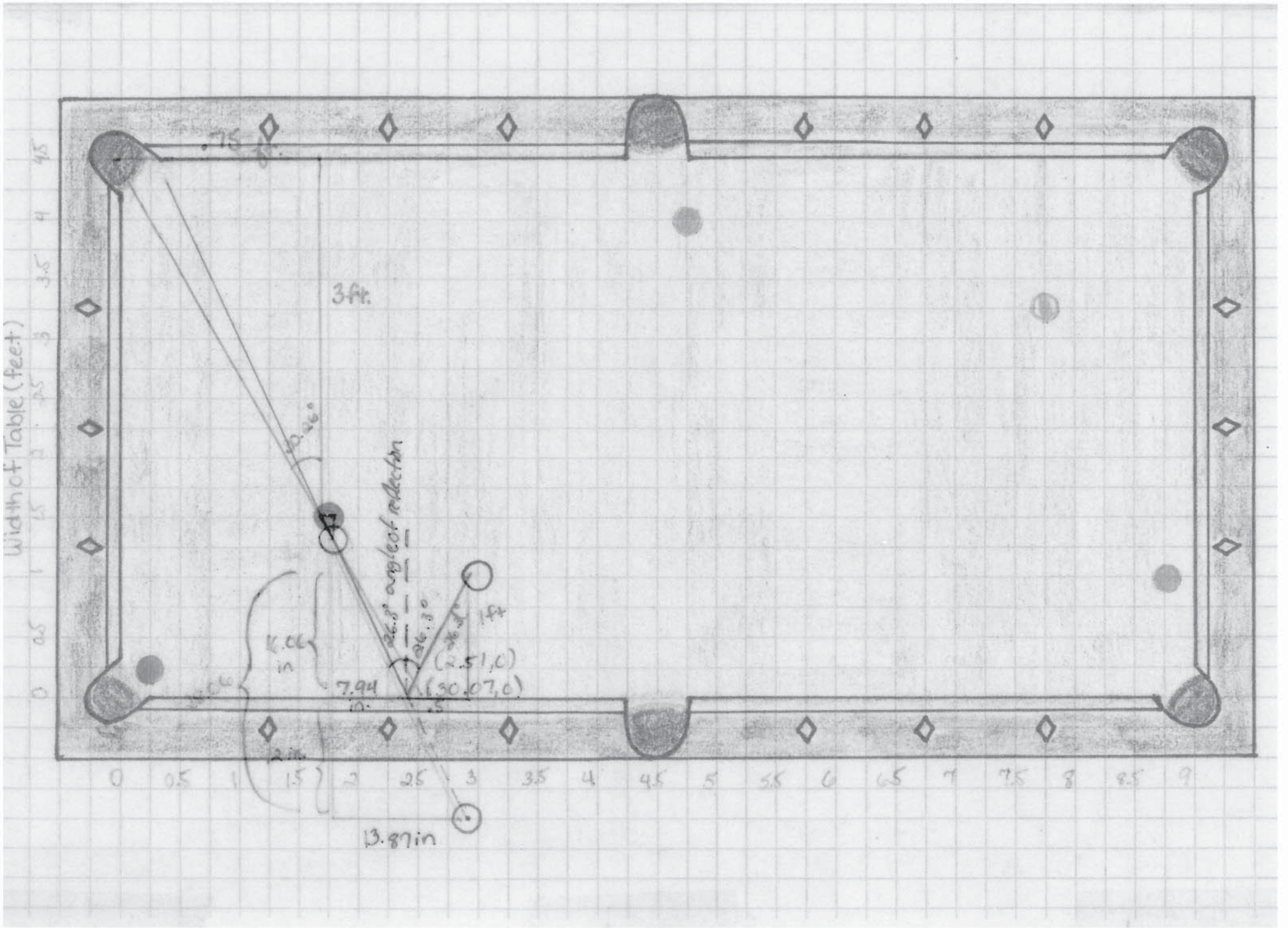
1.75 ft.×12 in. = 21 in.
21 in.+7.94 in.+1.13 in. = 30.07 in.
Convert **(30.07,0)** to feet
0 in.÷ 12 in. = 0 ft.
30.07 in.÷12 in. = 2.51 ft.
(2.51,0)

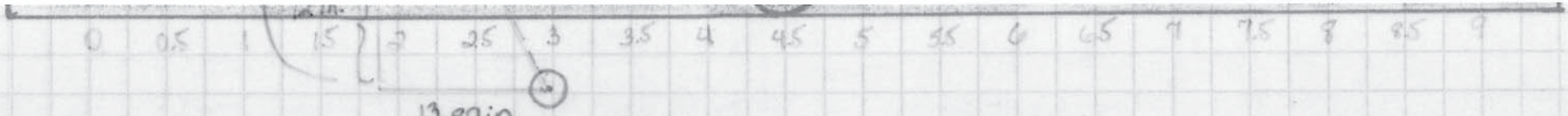
Angle of Reflection

Tan-1(28.06/13.87) = 63.70°
90°-63.70° = **26.3°**

Angle Cue Ball Is Struck At

The angle of reflection is 26.3° and the angle cue ball is struck at is equal to it. So the angle is **26.3°**


Width of Table (feet)
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
6
6.5
7
7.5
8
8.5
9
3ft.
26.6° angle of reflection
26.6°
26.6°
1ft
(2.5,1,0)
(30,07,0)
7.94 in
13.87in



Point of Contact on Rail

16.06 ÷ 12 = 28.06 in.

1.25 ft × 12in = 15in.

15in. - 1.13 in = 13.87 in.

$\frac{16.06}{28.06} = \frac{x}{13.87\text{ in.}}$

$\frac{222.7522}{28.06} = \frac{28.06x}{28.06}$

$x = 7.9384 - 24804$

$x = 7.94$ in.

.7564 × 12in. = 21 in.

21in. + 7.94in. = 30.07 in.

convert (30.07,0) to feet

0in. ÷ 12in. = 0 ft.

30.07in. ÷ 12in. = 2.51 ft.

(2.51, 0 ft.)

* Work in pink on different transparencies

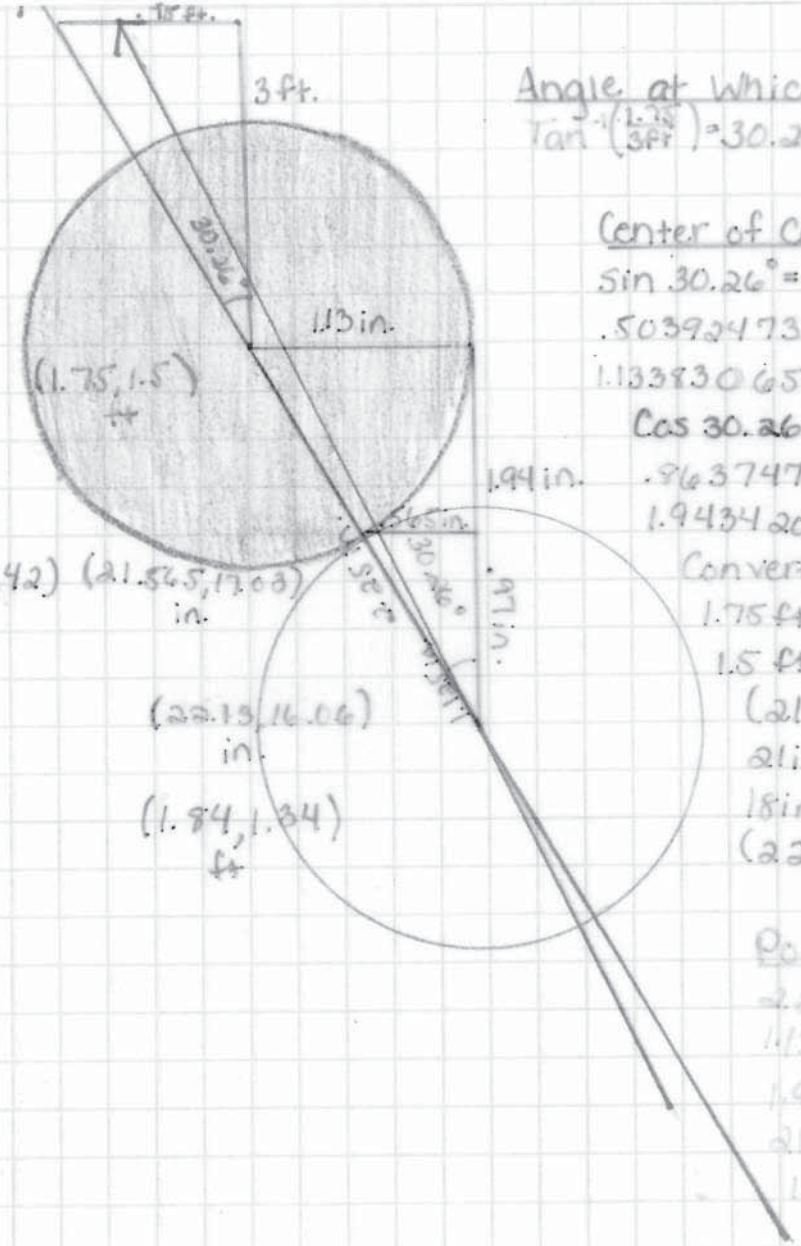
Angle Cue Ball is Struck At

The angle of reflection is 26.3° and the angle cue ball is struck at is equal to it. So the angle is 26.3°

Angle of Reflection

$\tan^{-1}\left(\frac{28.06}{13.87}\right) = 63.70°$

$90° - 63.70° = 26.3°$



Angle at Which Target Ball is Traveling

$\tan^{-1}\left(\frac{1.75}{3\text{ ft}}\right) = 30.26°$

Center of Cue Ball

$\sin 30.26° = \frac{a}{2.25}$

$.503924737 = \frac{a}{2.25}$

$1.133830658 = a \quad a = 1.13$ in.

$\cos 30.26 = \frac{b}{2.25}$

$.863747566 9 = \frac{b}{2.25}$

$1.9434 2026 = b \quad b = 1.94$

Convert (1.75, 1.5) to inches

1.75 ft. $\times 12$ in $= 21$ in.

1.5 ft $\times 12$ in. $= 18$ in.

$(21, 18)$ in.

21 in. $+ 1.13$ in. $= 22.13$ in.

18 in. $- 1.94$ in. $= 16.06$ in.

$(22.13, 16.06)$ in.

Point of Contact Between Cue Ball and Target

2.25 in. $\div 2 = 1.125$ in.

1.13 in. $\div 2 = .565$ in.

1.94 in. $- 2 = .97$ in.

21 in. $+ .565$ in. $= 21.565$ in.

18 in. $- .97$ in. $= 17.03$ in.

$(21.565, 17.03)$ in.

Measurements may not be to scale

Ming's Rail Shot

Ming's Rail Shot

Angle at Which Target Ball Travels At

$TAN^{-1} \frac{2}{2.5} = 38.65980825$

$TAN = 38.66^{\circ}$

Side A

$SIN\ 38.66^{\circ} = \frac{a}{2.25}$

$.6246976608 = \frac{a}{2.25}$

$(2.25)\ .6246976608 = \frac{a}{2.25}(2.25)$

$a = 1.405569737$

$a = 1.41$

Side B

$COS\ 38.66^{\circ} = \frac{b}{2.25}$

$.7808667188 = \frac{b}{2.25}$

$(2.25)\ .7808667188 = \frac{b}{2.25}(2.25)$

$b = 1.756950117$

$b = 1.76$

Center of Cue Ball at Contact

7*12=84in
84-1.41=82.59 in.

30+1.76=31.76 in
(82.59, 31.76) in

82.59/12=6.8825 ft.
31.76/12=2.65ft.
(6.8825, 2.65) ft.

Point of Contact Between Cue Ball and Target Ball

1.41/2=0.705 in
1.76/2=0.88 in
84-0.705= 83.295 in
30+0.88= 30.88 in
(83.295, 30.88) in

83.295/12= 6.94125 ft.
30.88/12= 2.57 ft.
(6.94125, 2.57) ft.

Height of Large Triangle

4.5*12=54in
54-31.76=22.24in

22.24+27=49.24in

Base Length of Larger Similar Triangle

4.75*12=57
82.59-57=25.59 in

Base Length of Smaller Similar Triangle

$\frac{27}{49.24} = \frac{x}{25.59}$

$\frac{690.93}{49.24} = \frac{49.24\,x}{49.24}$

x= 14.038865
x=14.03 in.

Point of Contact on Rail

$\frac{27}{49.24} = \frac{x}{25.59}$
$\frac{690.93}{49.24} = \frac{49.24}{49.24}$
14.0318865
14.03

4.75*12=57+14.03=71.03 in

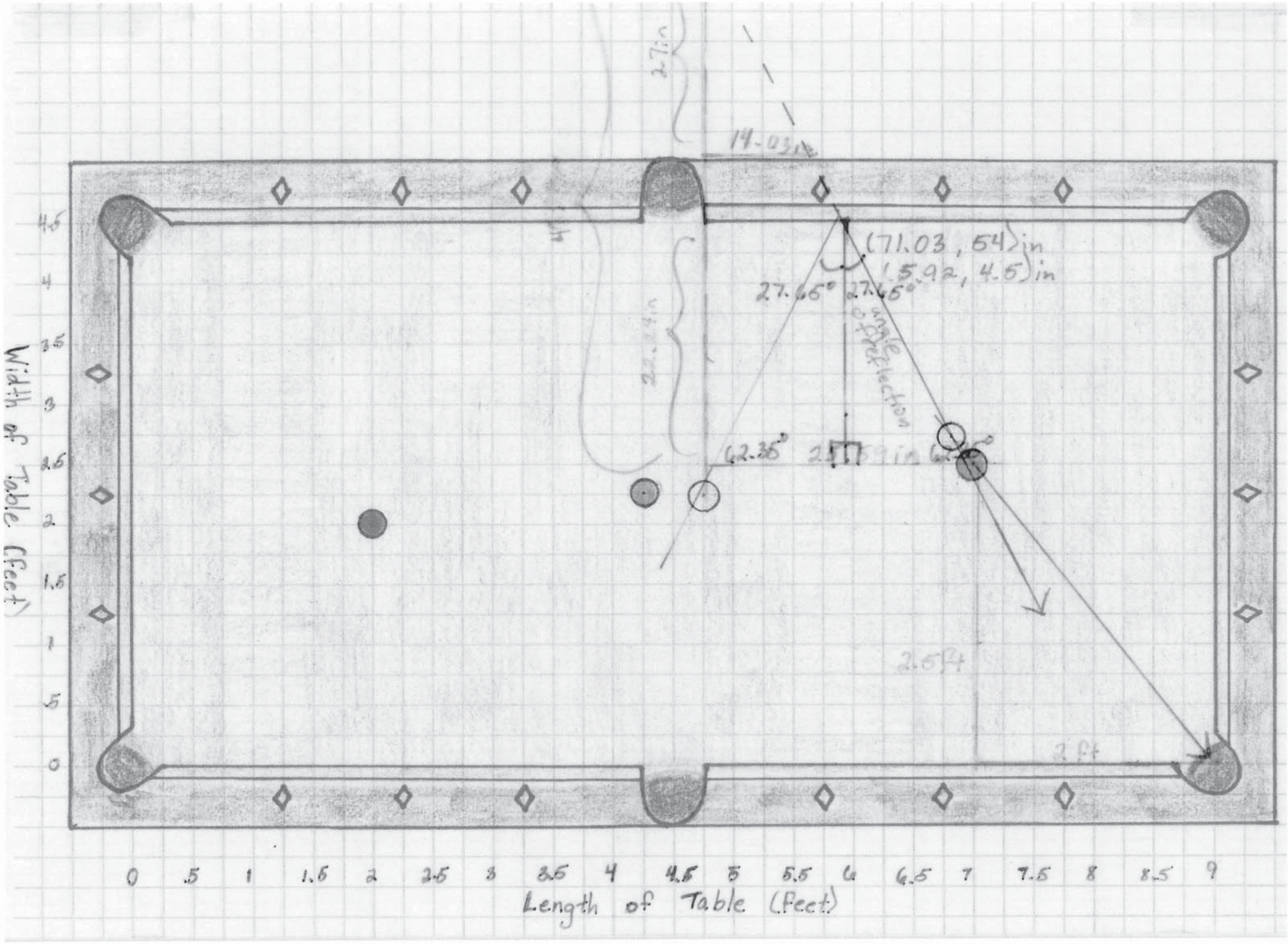
4.5*12=54 in
(71.03, 54) in
71.03/12=5.92
54/12=4.5
(5.92, 4.5) ft

Angle of Reflection

82.59-71.04=11.65
4.5*12=54
54-31.76=22.24
$TAN^{-1}\ \frac{11.65}{22.24} = 27.64693801$
$= 27.65°$

Angle at Which Cue Ball Travels At

90°-62.35°=27.65°


2.7in
14.03in
(71.03, 54)in
(59.2, 45)in
27.66°
27.66°
angle of reflection
62.36°
2.5ft
2ft
Width of Table (Feet)
4.5
4
3.5
3
2.5
2
1.5
1
.5
0
0 .5 1 1.5 2 2.5 3 3.5 4 4.5 5 5.5 6 6.5 7 7.5 8 8.5 9
Length of Table (Feet)

int of Contact on Rail

27 = x
.24 25.59

7 x 25.59 = x 49.24
90.93 = x 49.24
24 49.24
4.0318865
.03

75 x 12 = 57 + 14.03 = 71.03 in
5 x 12 = 54 in
71.03, 54) in
.03 ÷ 12 = 5.92
4 ÷ 12 = 4.5
5.92, 4.5) ft

* work for green on different transparencies

Angle of Reflection

82.59 - 71.04 = 11.65
4.5 x 12 = 54
54 - 31.76 = 22.24
TAN^{-1} 11.65 / 22.24 = 27.64693801°
≈ 27.65°

Angle At Wich Cue Ball Travels At

90° - 62.35 = 27.65°

Center of Cue Ball

$\tan^{-1}\frac{2}{2.25} = 38.659808\,25 \approx 38.66$

$\sin 38.66 = \frac{a}{2.25}$

$(2.25)(.624697\,6608) = \frac{a}{2.25}(2.25)$

$a = 1.405569737$

$a = 1.41$ in

$\cos 38.66 = \frac{b}{2.25}$

$(2.25)(.780864\,7188) = \frac{b}{2.25}(2.25)$

$7 \times 12 = 84$

$84 - 1.41 = 82.59$ in

$30 + 1.76 = 31.76$ in

$(82.59, 31.76)$ in

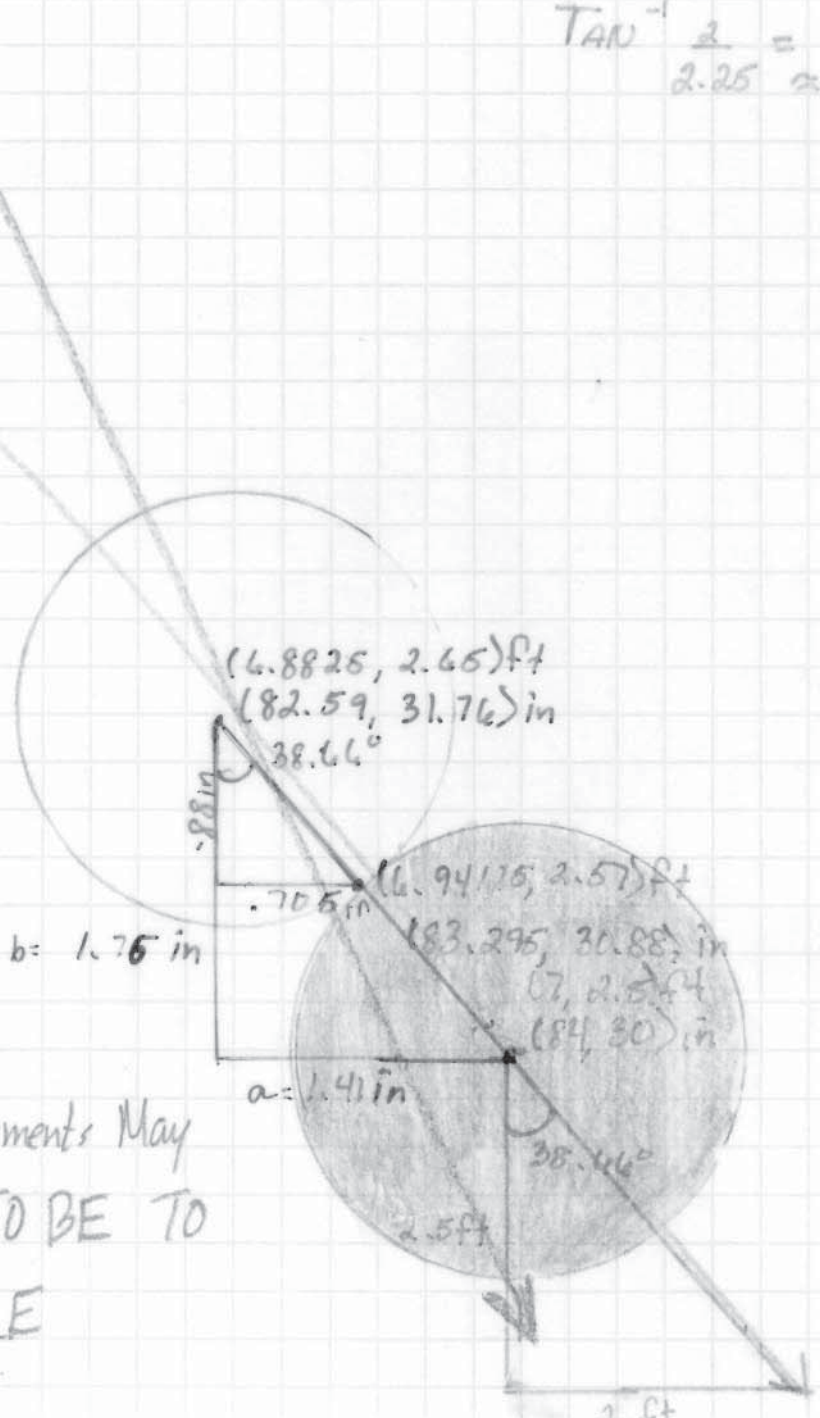
$82.59 \div 12 = 6.8825$ ft

$31.76 \div 12 = 2.65$ ft

$(6.8825, 2.65)$ ft

$b = 1.756980117$

$b = 1.76$ in



Measurements May NOT TO BE TO SCALE

Point of Contact Between Cue Ball and Target

$1.41 \div 2 = 0.705$

$1.76 \div 2 = 0.88$

$84 - 0.705 = 83.295$

$30 + 0.88 = 30.88$

$(83.295, 30.88)$ in

$83.295 \div 12 = 6.94125$

$30.88 \div 12 = 2.57$

$(6.94625, 2.57)$ ft

Isabela's Rail Shot

Isabela 's Rail Shot

Angle target ball is traveling at
$TAN^{-1}(1.5/2.5) = \mathbf{30.964^{\circ}}$

Center of Cue Ball
Measure of Side a
$SIN\ 30.964^{\circ} = a/\ 2.25$
$.5144993992 = a/\ 2.25$
$.5144993992 \times 2.25 = 1.157623648$
$\approx 1.16in = a$

Measure of Side *b*
$COS\ 30.964^{\circ} = b/2.25$
$.8574907394 = b/2.25$
$.8574907394 \times 2.25 = 1.929354164$
$\approx 1.93in = b$

Target ball location (3, 2) ft
3ft.×12, 2ft.×12 ←converting feet into inches
(36, 24) in
(36-1.16, 24-1.93)
(34.84, 22.07) in. ← Cue ball at contact with target ball

Point of Contact Between Cue Ball and Target Ball
1.16 in/ 2 = .58in
1.93/2= .965in
(36, 24) in
36-.58= 35.42in
24-.965=23.035
Convert **(35.42, 23.035) in** into feet
35.42/12=2.95ft.
23.035/12=1.92
(2.95, 1.92) ft

Point of Contact on Rail

To find the measure 58.07, I had to reflect the cue ball over the x-axis. The coordinate of the cue ball in the beginning is (1, 3) ft. or (12, 36) in. I drew down the line from the location of the cue ball at contact (34.84, 22.07) in. until it reached the same distance as the starting point of the cue ball reflected. Then I connected both points making two similar triangles. The dimensions for the larger triangle is 58.07in.= height, and 22.84in.= width.

Length of x

$$\frac{22.07}{58.07} = \frac{x}{22.84\text{in.}}$$

$22.07 \times 22.84 = 504.0788$

$504.0788 \div 58.07 = 8.680537283$

$x = 8.680537283$

$x \approx$ **8.68in**

Point that Cue ball must hit

$34.84 - 8.68 = 26.16$

(26.16,0)in.

$26.16 \div 12 = 2.18$ft.

(2.18, 0)ft.

Angle of Reflection

$\text{Tan}^{-1}(22.07 \div 8.68)$

$68.5305802 \approx 68.53°$

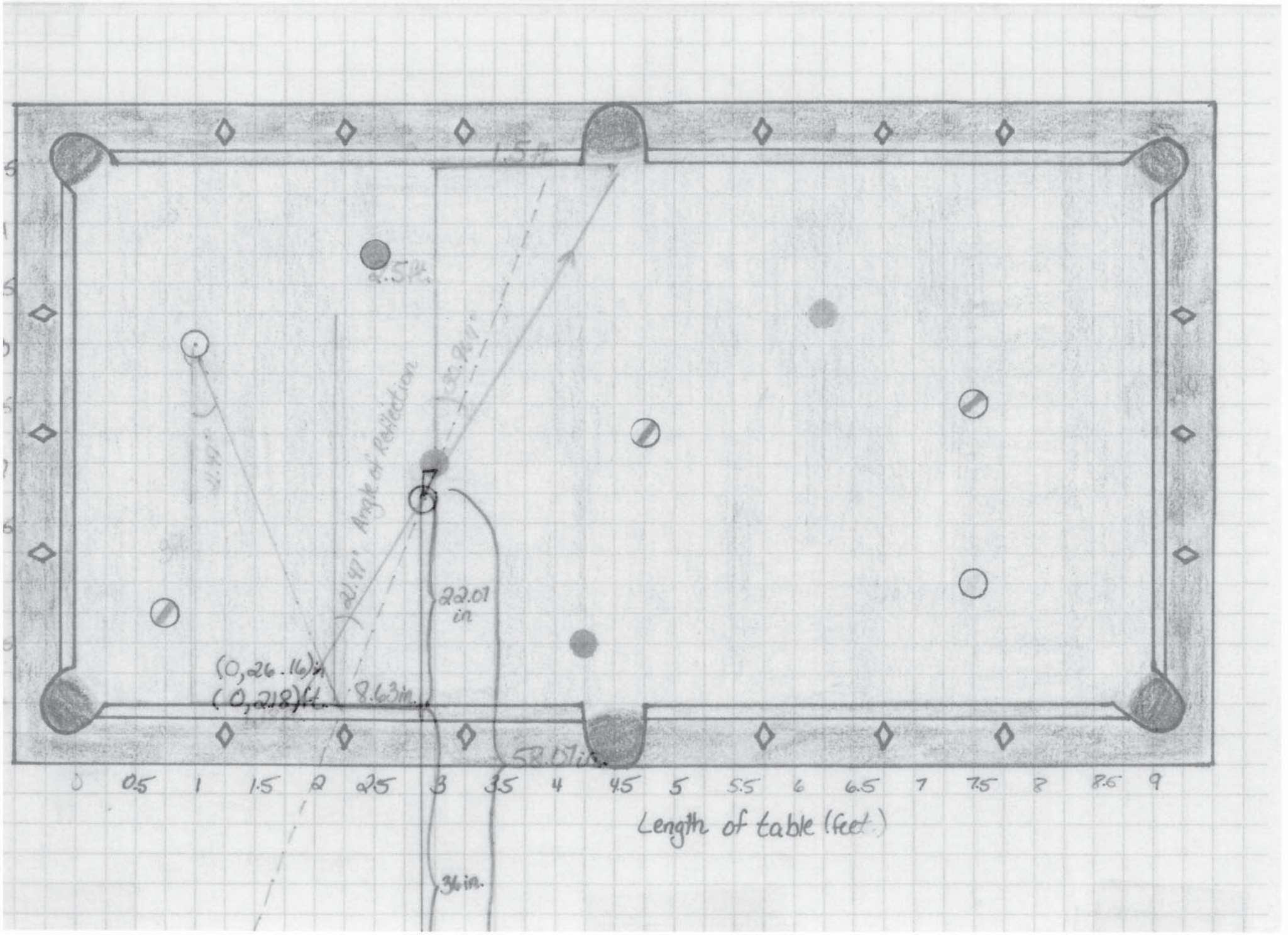
$90° - 68.53° =$ **21.47°**

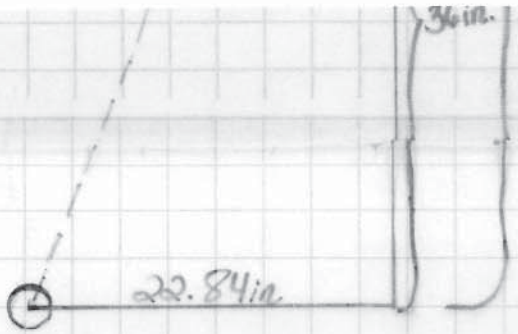
Angle of reflection ↑

Angle Cue Ball is Traveling

$90° + 68.53° = 158.53°$

$180° - 158.53° =$ **21.47°**


1.5 ft
2.5 ft
32.94°
22.41° Angle of Reflection
22.01 in
(0,26.16) in
(0,2.18) ft
8.63 in
52.07 in
36 in
0 0.5 1 1.5 2 2.5 3 3.5 4 4.5 5 5.5 6 6.5 7 7.5 8 8.5 9
Length of table (feet)



Point of Contact on Rail

To find the measure 58.07in. I had to reflect the cue ball over the x-axis. The coordinate of the cue ball in the beginning is (1,3)ft. or (12,36)in. I drew down the line from the location of the cue ball at contact (34.24, 22.07)in. until it reached the same distance as the starting point of the cue ball reflected. Then I connected both points making two similar triangles. The dimensions for the larger triangle is 58.07in. = height, and 22.84 in. = width.

Length of x:

$$\frac{22.07}{58.07} = \frac{x}{22.84\text{in.}}$$

$22.07 \times 22.84 = 504.0788$

$504.0788 \div 58.07 = 8.680537283$

$x = 8.680537283$

$x \approx 8.68$ in.

Point that Cue ball must hit

$34.84 - 8.68 = 26.16$ in.

(26.16, 0)in.

$26.16 \div 12 = 2.18$ ft.

(0, 2.18)ft.

Angle of Reflection

$\tan^{-1}(22.07 \div 8.68)$

$= 68.53058022 \approx 68.53°$

$90° - 68.68° = 21.47°$

↑ Angle of reflection

Angle Cue Ball is Traveling

$90° + 68.53° = 158.53°$

$180° - 158.53° = 21.47°$

* Work on another transparency.

Angle at which Target Ball is Traveling

$\tan^{-1}(1.5/2.5) = 30.964°$

Center of Cue Ball

$\sin 30.964° = \frac{a}{2.25}$

$.514493992 = \frac{a}{2.25}$

$1.157611482 = a$

$1.16 = a$

$\cos 30.964° = \frac{b}{2.25}$

$.857497394 = \frac{b}{2.25}$

$1.928118364 = a$

$1.93 = a$

Convert (3, 2) ft

3ft x 12 = 36 in

2ft x 12 = 24 in

(36, 24) in

36 in - 1.16 in = 34.84 in

24 in - 1.93 in = 22.07 in

(34.84, 22.07) in

Point of Contact Between Cue Ball and Target

1.16 in ÷ 2 = .58 in

1.93 in ÷ 2 = .965 in

(36, 24) in

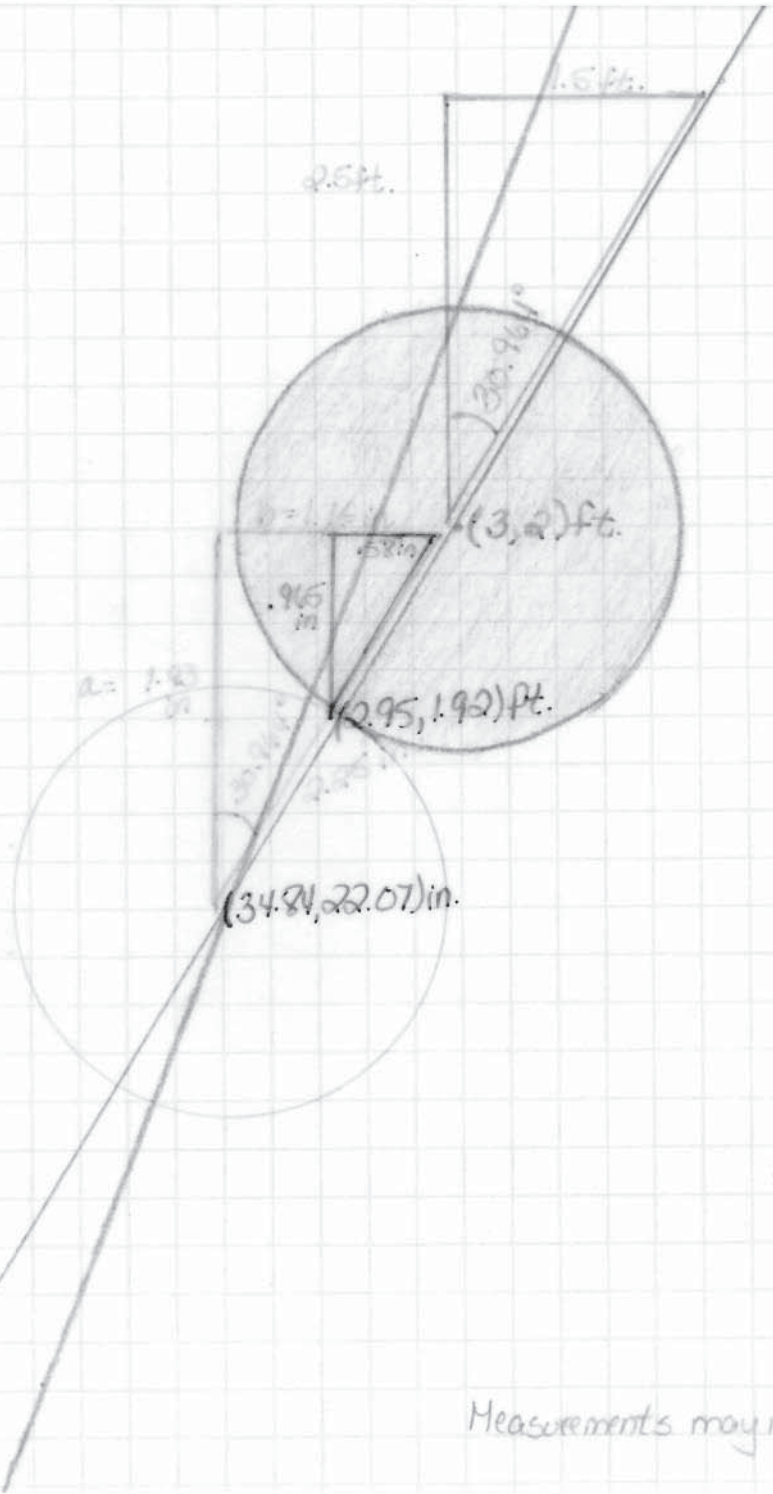
36 in - .58 in = 35.42 in

24 in - .965 in = 23.035 in

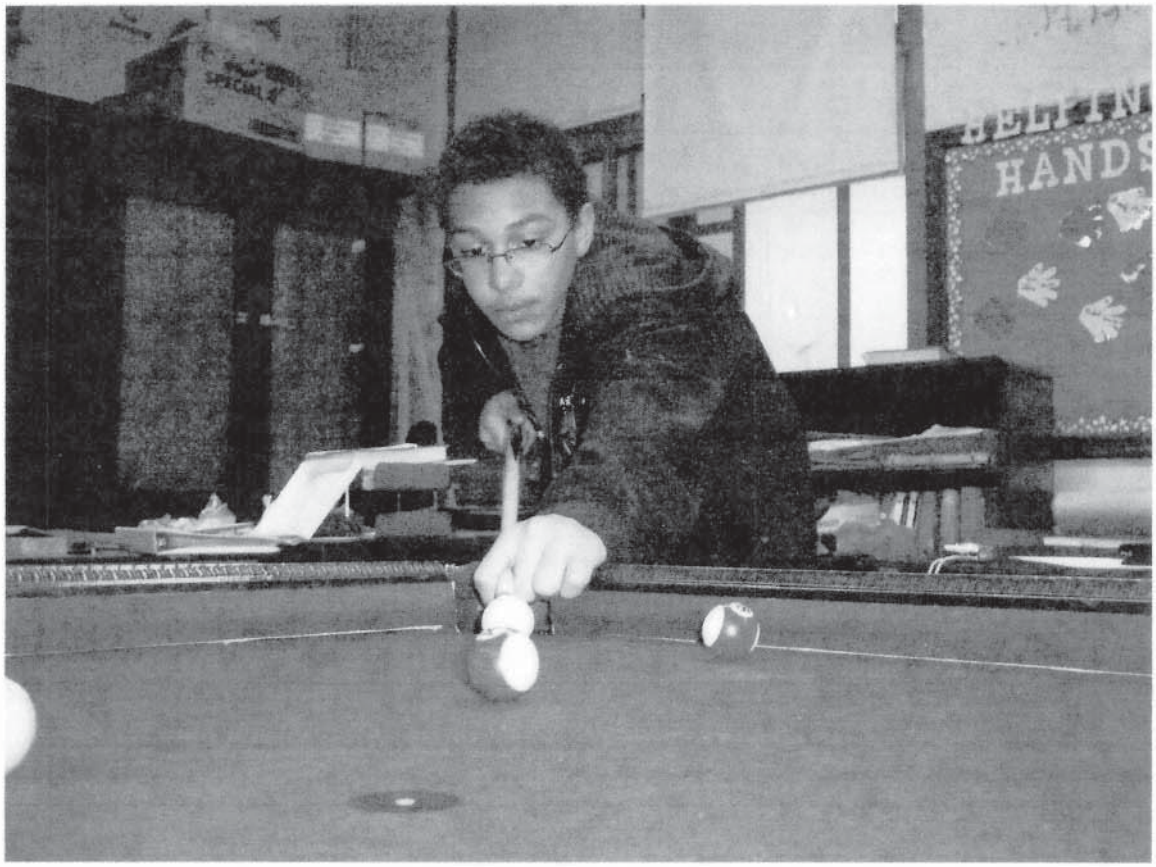
Convert (35.42, 23.035) in into feet

35.42 in ÷ 12 = 2.95 ft. (2.95, 1.92) ft.

23.035 in ÷ 12 = 1.92 ft.



Measurements may not be to scale



Darius's Rail Shot

Darius's Rail Shot

Measure of Angle Target Ball Travels In

$$TAN^{-1}(x) = \frac{3.25}{2}$$

$x \approx 58.39^{o}$

Travel Distance of Target Ball

$a^2 + b^2 = c^2$

$24\ in^2 + 39\ in^2 = c^2$
$576\ in + 1{,}521\ in = c^2$
$c^2 = 2{,}097\ in$
$c = 45.79301257\ in \approx 45.79\ in/12 = 3.81583 \approx 3.82 ft$

Scale Factor From Large Triangle to Small Triangle

$2.25\ in/45.79\ in = .0491373662$
$.0491373662 * 24\ in = 1.17929679\ in \approx 1.18\ in$ side a'
$.0491373662 * 39\ in = 1.916357282\ in \approx 1.92\ in$ side b'

Center of Cue Ball at Point of Contact

$84\ in - 1.18\ in = 82.82\ in/12 \approx 6.90\ ft$

$15\ in - 1.92\ in = 13.08\ in/12 = 1.09\ ft$

$(6.90\ ft, 1.09\ ft)$
$(82.82\ in, 13.08\ in)$

Point of Contact Between Cue Ball and Target Ball

$1.18 \text{ in}/2 = .59 \text{ in}$
$1.92 \text{ in}/2 = .96 \text{ in}$

$15 \text{ in} - .96 \text{ in} = 14.04 \text{ in} = 1.17 \text{ ft}$
$82.82 \text{ in} + .59 \text{ in} = 83.41 \text{ in}/12 = 6.95 \text{ ft}$

$(6.95 \text{ ft}, 1.17 \text{ ft})$
$(83.41 \text{ in}, 14.04 \text{ in})$

Point of Rail Contact by Cue Ball

The distance from the cue ball to the rail is 24 in. I reflected that over the axis and got part of my triangle. The distance from the cue ball at contact with target ball to the rail is 13.08 in. When you add these 2 measurements it equals 37.08 in, which is the height of the triangle. The base of the triangle is the length from the cue ball to the target ball, which is 28.82 in.

$$\frac{13.08 \text{ in}}{37.08 \text{ in}} = \frac{x}{28.82 \text{ in}}$$

$28.82 * 13.08/37.08 = 10.06045307 \text{ in} = x$
$82.82 \text{ in} - 10.06045307 \text{ in} = 72.65372168 \text{ in}/12 \approx 6.05 \text{ ft}$

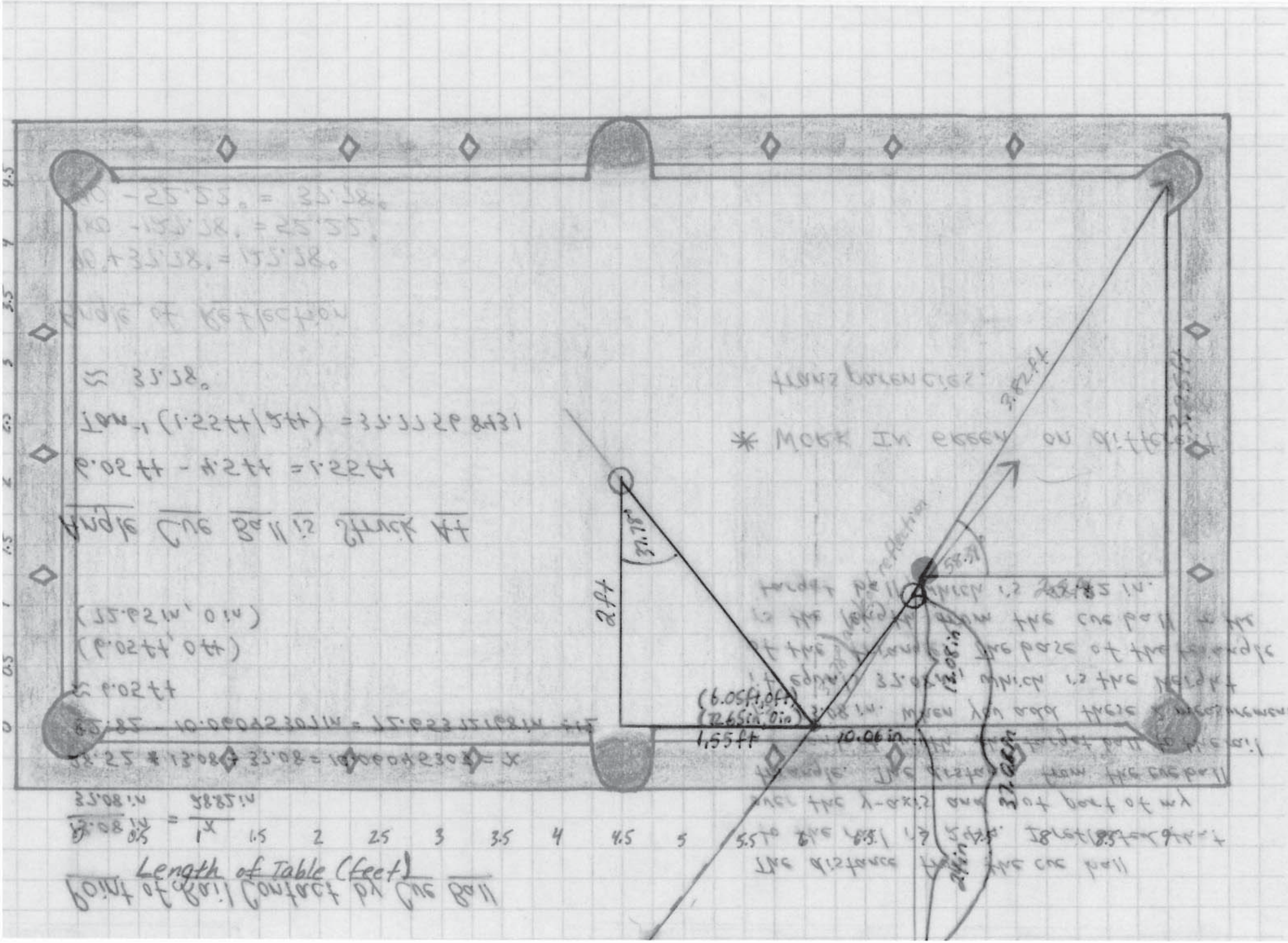
$(6.05 \text{ ft}, 0 \text{ ft})$
$(72.65 \text{ in}, 0 \text{ in})$

Angle Cue Ball is Struck At

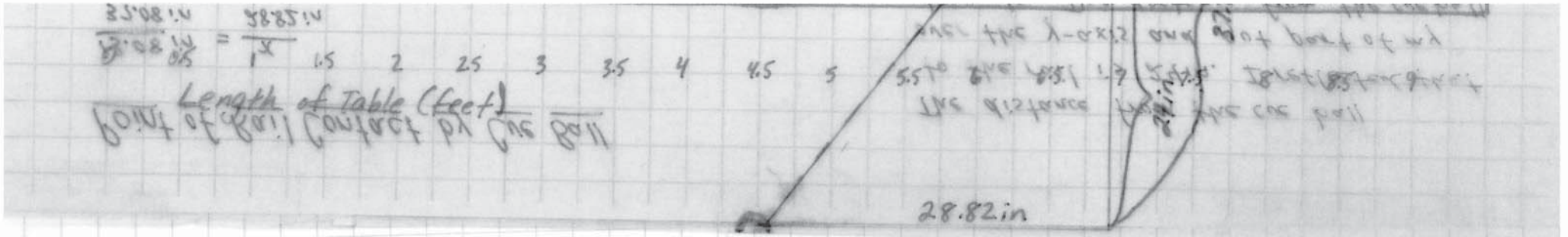
$6.05 \text{ ft} - 4.5 \text{ ft} = 1.55 \text{ ft}$
$TAN^{-1}(1.55 \text{ ft}/2 \text{ ft}) = 37.77568431^{\circ} = 37.78^{\circ}$

Angle of Reflection

$90^{\circ} + 37.78^{\circ} = 127.78^{\circ}$
$180^{\circ} - 127.78^{\circ} = 52.22^{\circ}$
$90^{\circ} - 52.22^{\circ} = 37.78^{\circ}$



2ft
37.78°
(6.05ft, 0ft)
(12.65in, 0in)
1.55ft
10.06in
13.08in
32.08in
24in
3.82ft
reflection
0 0.5 1 1.5 2 2.5 3 3.5 4 4.5 5 5.5 6 6.5 7 7.5 8 8.5 9
Length of Table (feet)


1.5
2
2.5
3
3.5
4
4.5
5
28.82in

Measurement of Angle Targetball travels at

$\tan^{-1}(3.25/2) = x$

$x = 58.39°$

Travel Distance of Target Ball

$a^2 + b^2 = c^2$

$24in^2 + 39in^2 = c^2$

$576 + 1,521 = c^2$

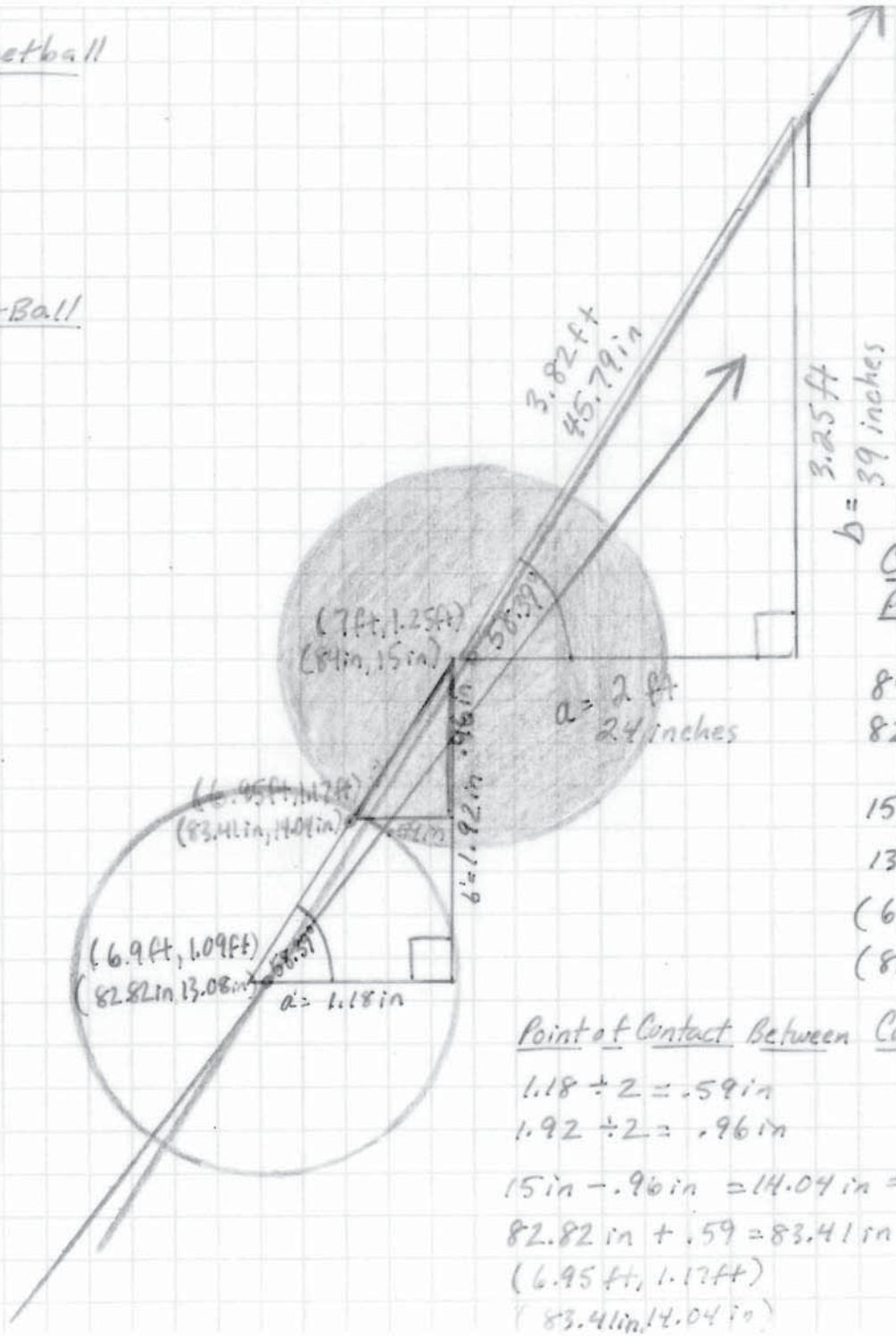
$2,097 = c^2$

$45.79301257 = c$

$45.79in \approx c$

$45.79 \div 12 \approx$

$3.82ft$



Scale Factor From Large Triangle to Small Triangle

$2.25in / 45.79 = 0.491373662$

$0.491373662 \times 24 =$

$1.17929679 \approx 1.18in$

$0.491373662 \times 39 =$

$1.916357282 \approx 1.92in$

Center of Cue Ball at Point of Contact

$84in - 1.18in = 82.82in$

$82.82 \div 12 \approx 6.9ft$

$15in - 1.92 = 13.08in$

$13.08 \div 12 \approx 1.09$

$(6.9ft, 1.09ft)$

$(82.82in, 13.08in)$

Point of Contact Between Cue Ball and Target Ball

$1.18 \div 2 = .59in$

$1.92 \div 2 = .96in$

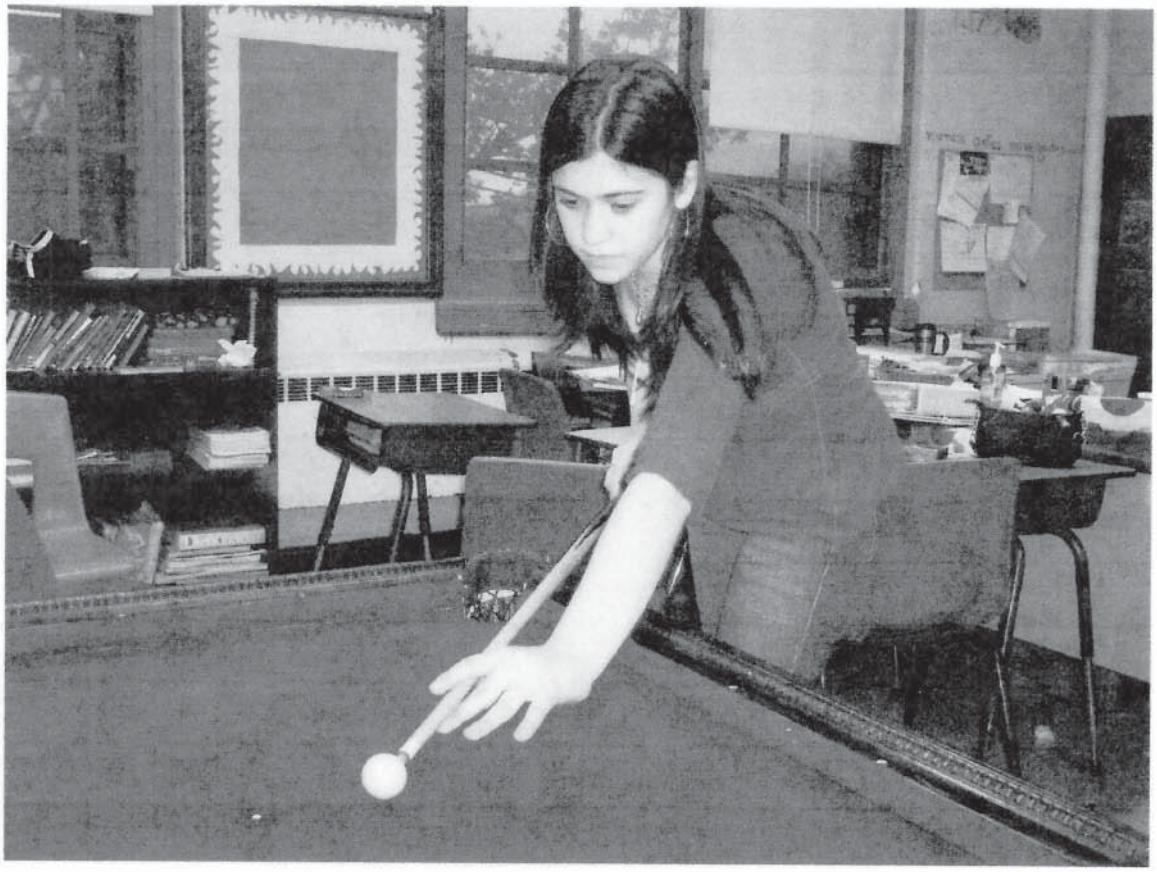
$15in - .96in = 14.04in = 1.17ft$

$82.82in + .59 = 83.41in \div 12 = 6.95ft$

$(6.95ft, 1.17ft)$

$(83.41in, 14.04in)$

Measurements may NOT be to scale



Catherine's Rail Shot

Catherine's Rail Shot

Angle at Which Cue Ball is Traveling At

$\tan^{-1}(1.25/1.5) = 39.80557109$

$\approx$ **39. 81°** → the angle at which the target ball enters pocket

Center of Cue Ball

Sin 39.81 = b/2.25

(2.25) .6402437805 = b/2.25 (2.25)

b = 1.440548506

≈ 1.44 in.

Cos 39.81 = a/2.25

(2.25) .7681717917 = a/2.25 (2.25)

a = 1.728386531

≈ 1.73 in.

(3.25,1.5) ft.

x 12

(39, 18) in.

39- 1.44 = 37.56 in.

18 + 1.73 = 19.73 in.

(37.56, 19.73) in.

/12

(3.13, 1.64) ft. → point where the center of cue ball is at

Point of Contact Between Cue Ball and Target Ball

1.73in. / 2 =.865 in.

1.44in. / 2 = .72 in.

(39,18)

39- .72 = 38.28

18+ .865 = 18.865

(38.28,18.865) in.

/12

(3.19, 1.57) ft. → point where cue ball and target ball meet/contact

Point on Rail that Cue Ball is Aimed At

Large triangle

3.13- .75 = 2.38 ft. → the base of large triangle

4.5- 1.64 = 2.86 ft.

2.86 ft. + 1ft. = 3.86 ft. → the height of large triangle

Small triangle

$\frac{1}{3.86} = \frac{x}{2.38}$

$\frac{2.38}{3.86} = \frac{3.86x}{3.86}$

x = .62 ft.→ the width of small triangle

1 ft. → the length of small triangle

Point at Which Cue Ball Hits the Rail

.75 + .62 = 1.37 ft.

(1.37, 4.5) ft.

x 12

(16.44, 54) in.

↑

point cue ball is aimed at

Angle of Reflection

$\tan^{-1}$(12in/ 7.44in) = 58.20°

90°- 58.20° = **31.8°** → angle at which the cue ball reflects

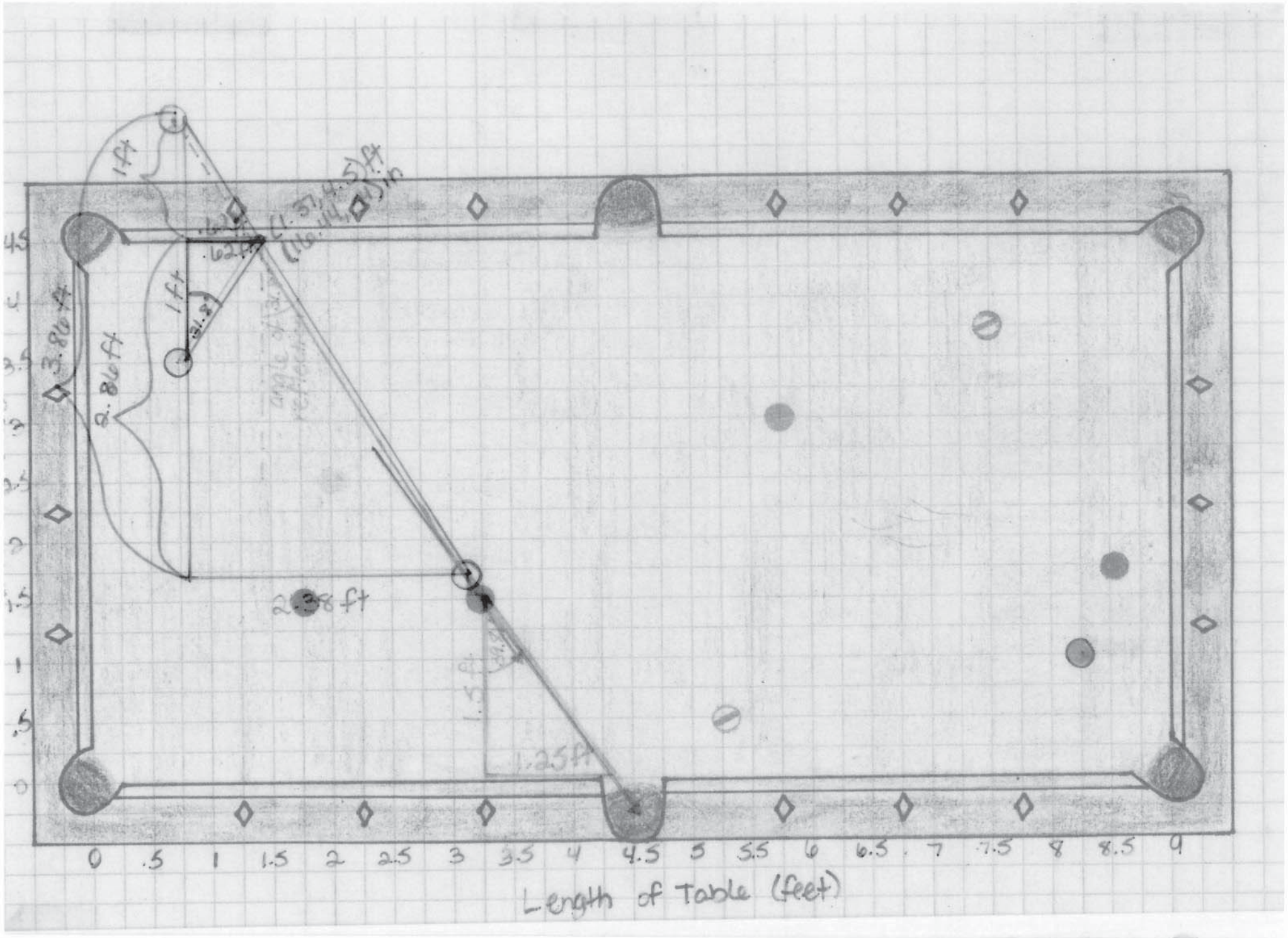
Angle That Cue Ball is Struck At

58.2° + 90 = 148.2°

180- 148.2° = **31.8°**

↑

Cue ball is struck at this angle


1ft
.62ft
(1.37, 4.5)ft
(16, 44, 4)in
1ft
(31.8)
3.86 ft
2.86 ft
2.38 ft
1.5 ft
1.25ft
0 .5 1 1.5 2 2.5 3 3.5 4 4.5 5 5.5 6 6.5 7 7.5 8 8.5 9
Length of Table (feet)

Point on Rail that Cue Ball is Aimed At

Large triangle

.13 - .75 = 2.38 ft. → the base of large triangle

4.5 - 1.64 = 2.86 ft

.86 ft. + 1 ft. = 3.86 ft → the height of large triangle

Small Triangle

$\frac{1}{3.86} = \frac{x}{2.38}$

$\frac{2.38}{3.86} = \frac{8.81x}{3.86}$

$x = .62$ ft → the base of small triangle

1 ft → the height of small triangle

Point at which cue ball hits rail

.75 + .62 = 1.37 ft.

(1.37, 4.5) ft

x 12

(16.44, 54) in

Angle that cue ball is struck

58.2° + 90° = 148.2°

180 - 148.2° = 31.8°

angle cue ball is struck at

Angle of Reflection

$\tan^{-1}\left(\frac{2.38}{7.44 \text{ in}}\right) = 58.2°$

90° - 58.20° = 31.8°

angle at which the cue ball reflects

*work in green on different transparencies

Angle at Which Target Ball is Traveling At

$\tan^{-1}\left(\frac{1.25}{1.5}\right) = 39.80557109$

$\approx 39.81°$

Center of Cue Ball

$\sin 39.81 = \frac{x}{2.25}$

$2.25)\ .640243705 = \frac{x}{2.25}(2.25)$

$x = 1.440548506$

≈ 1.44 in

$\cos 39.81 = \frac{x}{2.25}$

$2.25)\ .768171791 7 = \frac{x}{2.25}(2.25)$

$x = 1.728386531$

≈ 1.73 in

$(3.25, 1.5)$ ft

$\times 12$

$(39, 18)$ in

$39 - 1.44 = 37.56$ in

$18 + 1.73 = 19.73$ in

$(37.56, 19.73)$ in

$\div 12$

$(3.13, 1.64)$ ft

Point of Contact Between Cue Ball and Target Ball

$1.73 \text{ in} \div 2 = .865$ in

$1.44 \text{ in} \div 2 = .72$ in

$(39, 18)$ in

$39 - .72 = 38.28$

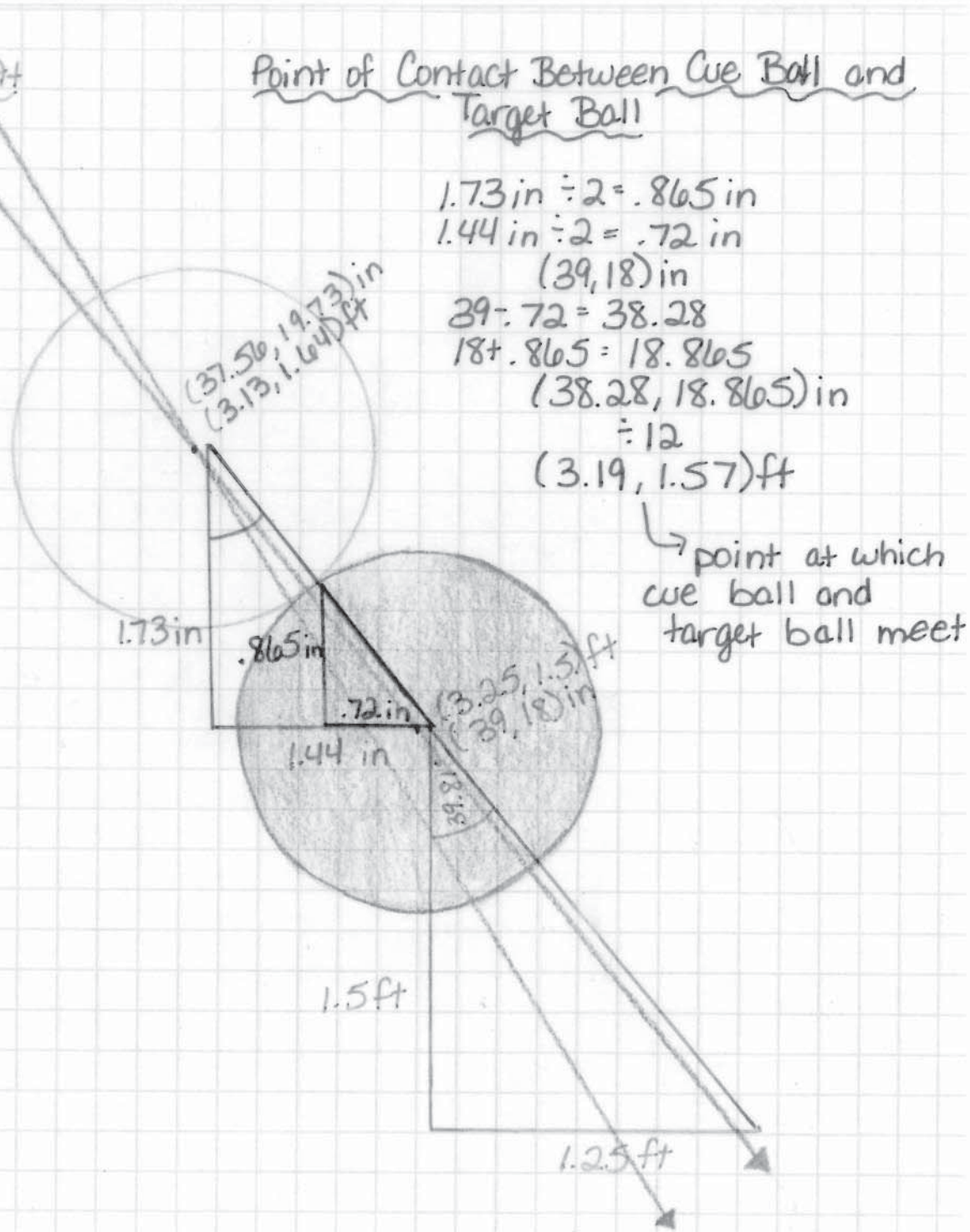
$18 + .865 = 18.865$

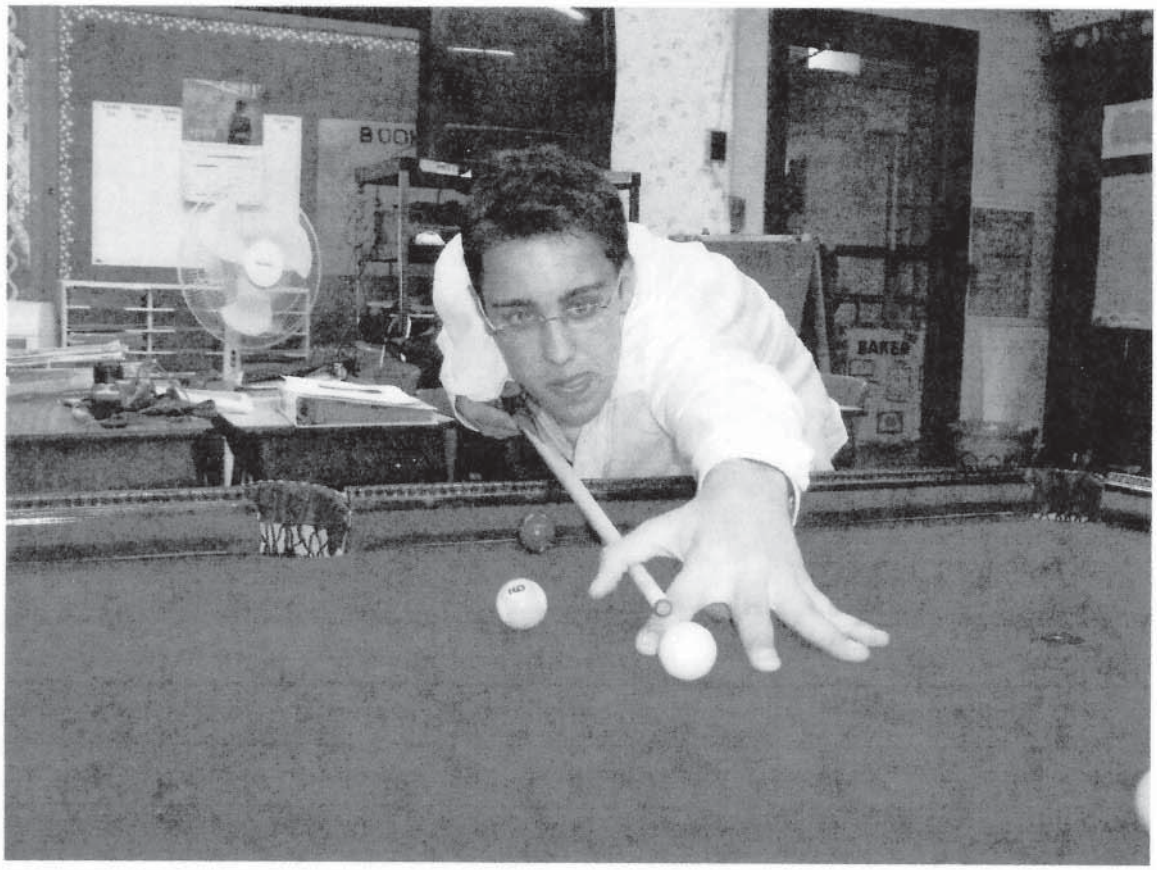
$(38.28, 18.865)$ in

$\div 12$

$(3.19, 1.57)$ ft

↳ point at which cue ball and target ball meet





Luis's Rail Shot

Luis's Rail Shot

Angle Target Ball must be hit at

$\text{Tan}^{-1}(1/.75) = \mathbf{53.130102235^\circ \approx 53.13^\circ}$

$90^\circ - 53.130102235^\circ = \mathbf{36.86989777^\circ \approx 36.87^\circ}$

Center of Cue Ball at Contact

Measure of Side *x*

$\text{Sin } 53.13010235^\circ = {}^{x}/_{2.25}$ inches

$(2.25).8 = {}^{x}/_{\cancel{2.25}(2.25)}$

$x = \mathbf{1.8}$ **inches**

$1.8 \div 12 = .15$ feet

$x = \mathbf{.15}$ **feet**

Measure of Side *a*

$\text{Cos } 53.13010\ 235^\circ = {}^{a}/_{2.25}$ inches

$(2.25).6000000001 = {}^{a}/_{\cancel{2.25}(2.25)}$

$a = \mathbf{1.35}$ **inches**

$1.35 \div 12 = .1125$ inches

$a = \mathbf{.1125}$ **feet**

Coordinate of Cue Ball at Contact with Target

(8, 3.75)

$8 \times 12 = 96$ inches

$3.75 \times 12 = 45$ inches

$(96 - 1.8, 45 - 1.35)$

(94.2inches, 43.65 inches)

$43.65 \div 12 = 3.6375$ feet

$94.2 \div 12 = 7.85$ feet

(7.85 feet, 3.6375 feet)

Point of Contact between Cue Ball and Target Ball

$.15 \div 2 = .075$ feet

$8 - .075 = 7.925$ feet

$.1125 \div 2 = .05625$ feet

$3.75 - .05625 = 3.69375$ feet ≈ 3.69 feet

(7.925 feet, 3.69375 feet)

$7.925 \times 12 = 95.1$ inches

$3.69375 \times 12 = 44.325$ inches

(95.1 inches, 44.325 inches)

Cue Ball Point of Contact on Rail

Measure of Side *z*

$3.6375 + 4.25 = 7.8875$ feet

$2 - .15 = 1.85$ feet

$z =$ **1.85 feet**

$1.85 \times 12 = 22.2$ inches

$z =$ **22.2 inches**

Measure of Side *y*

$\frac{3.6375}{7.8875} = \frac{y}{1.85}$

$(1.85)\ .4616116751 = y/\cancel{1.85(1.85)}$

$y =$ **.853981599 feet ≈ .85 feet**

$.853981599$ feet $\times 12 = 10.24777919$ inches

$y =$ **10.25 inches**

Rail Point of Contact

$.15 + .853981599 = 1.003981599 \approx 1$ foot

$8 - 1.003981599 = 6.996018401 \approx 7$ feet

(6.996018401 feet, 0 feet)

$6.996018401 \times 12 = 83.95222081$ inches ≈ 83.95 inches

$0 \times 12 = 0$ inches

(83.95222081 inches, 0 inches)

Angle at which Cue Ball Travels

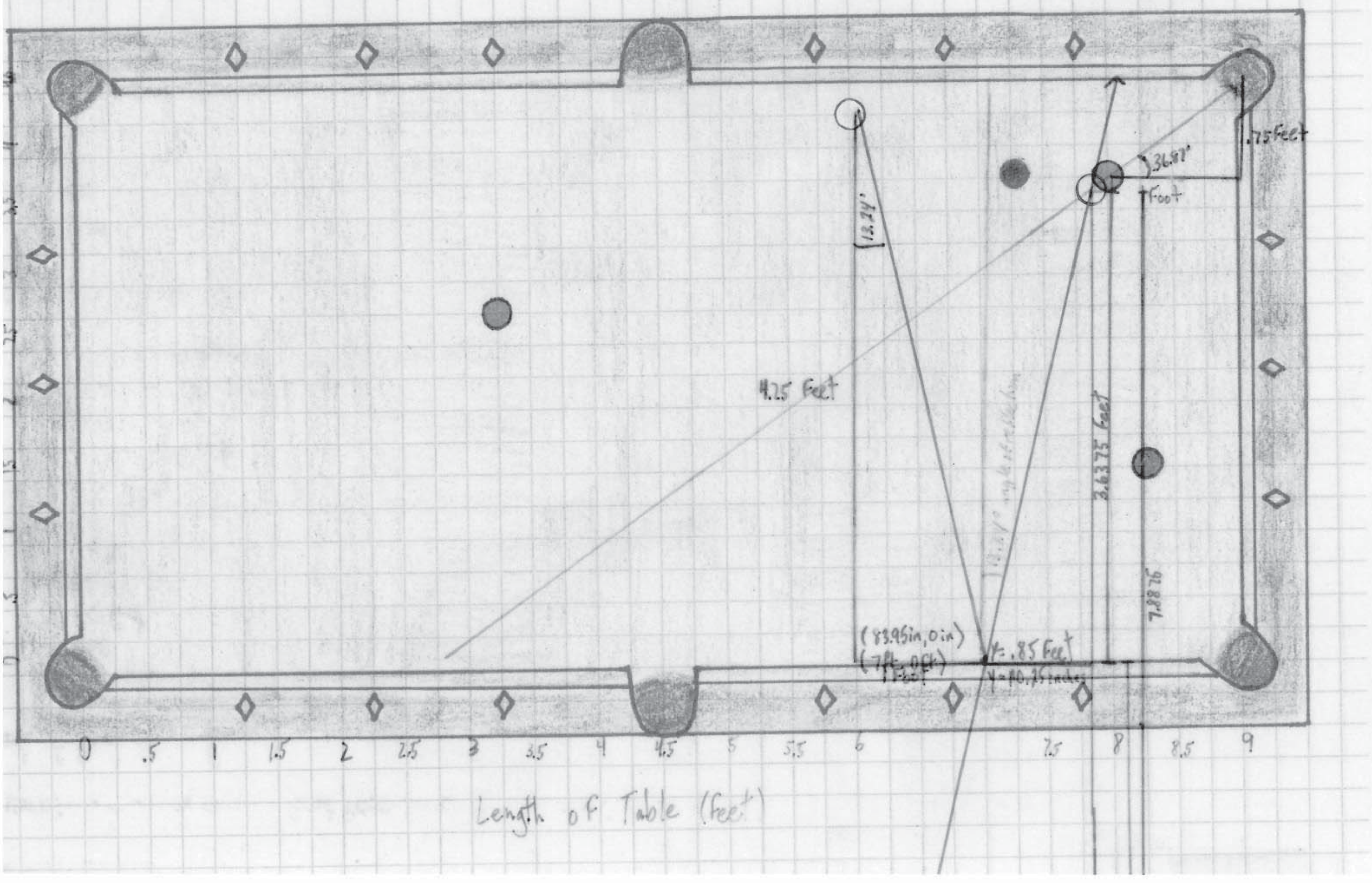
$90° + 13.24051992° = 103.24051992° \approx 103.24°$

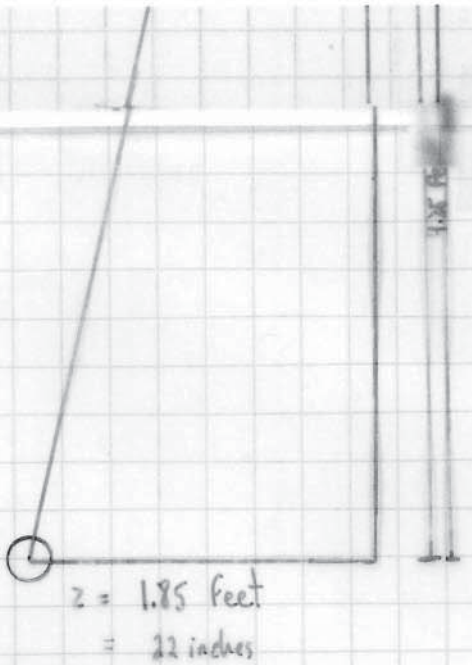
$180° - 103.24051992° = 76.75948008° \approx 76.76°$

$90° - 76.75948008° =$ **13.24051992° ≈13.24°**

Angle of Reflection

$\text{Tan}^{-1}(\frac{1}{4.25}) =$ **13.24051992° ≈ 13.24°**


.75 feet
36.87°
Foot
13.24°
4.25 Feet
3.6375 Feet
7.8875
(83.95in, 0in)
y = .85 Feet
0
.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
6
7.5
8
8.5
9
Length of Table (feet)



* work in blue shown on a different transparency

Point of Contact on Rail

3.1375 + 4.25 = 7.3875 feet
2 - .15 = 1.85 feet
z = 1.85 feet
1.85 x 12 = 22.2 inches
z = 22.2 inches

3.1375 / 7.3875 = Y / 1.85
(1.85) .4661116751 = Y / 1.85 (1.85)
y = .8539815 99 feet
.8539815 99 x 12 = 10.24777919 inches
y = 10.24777919 inches

.15 + .8539815 99 = .0039815 99 Feet
8 - 1.0039815 99 = 6.9960184 01 Feet
(6.9960184 01 Feet, 0 Feet)
6.9960184 01 x 12 = 83.95222081 inches
0 x 12 = 0 inches
(83.95222081 inches, 0 inches)

Angle Cue Ball is Struck at

90° + 13.24051992° = 103.24051992°
120° - 103.24051992° = 76.75948008°
90° - 76.75948008° = 13.24051992°

Angle of Reflection

Tan⁻¹(1/4.25) = 13.24051992°

Angle Target Ball is Traveling at

$\tan^{-1}(1/.25) = 53.13010235°$

$90° - 53.13010235° = 36.86989777°$

Center of Cue Ball

$\sin 53.13010235° = x/2.25$ in

$(2.25).8 = x/2.25(2.25)$

$x = 1.8$ inches

$1.8 \div 12 = .15$ Feet

$x = .15$ Feet

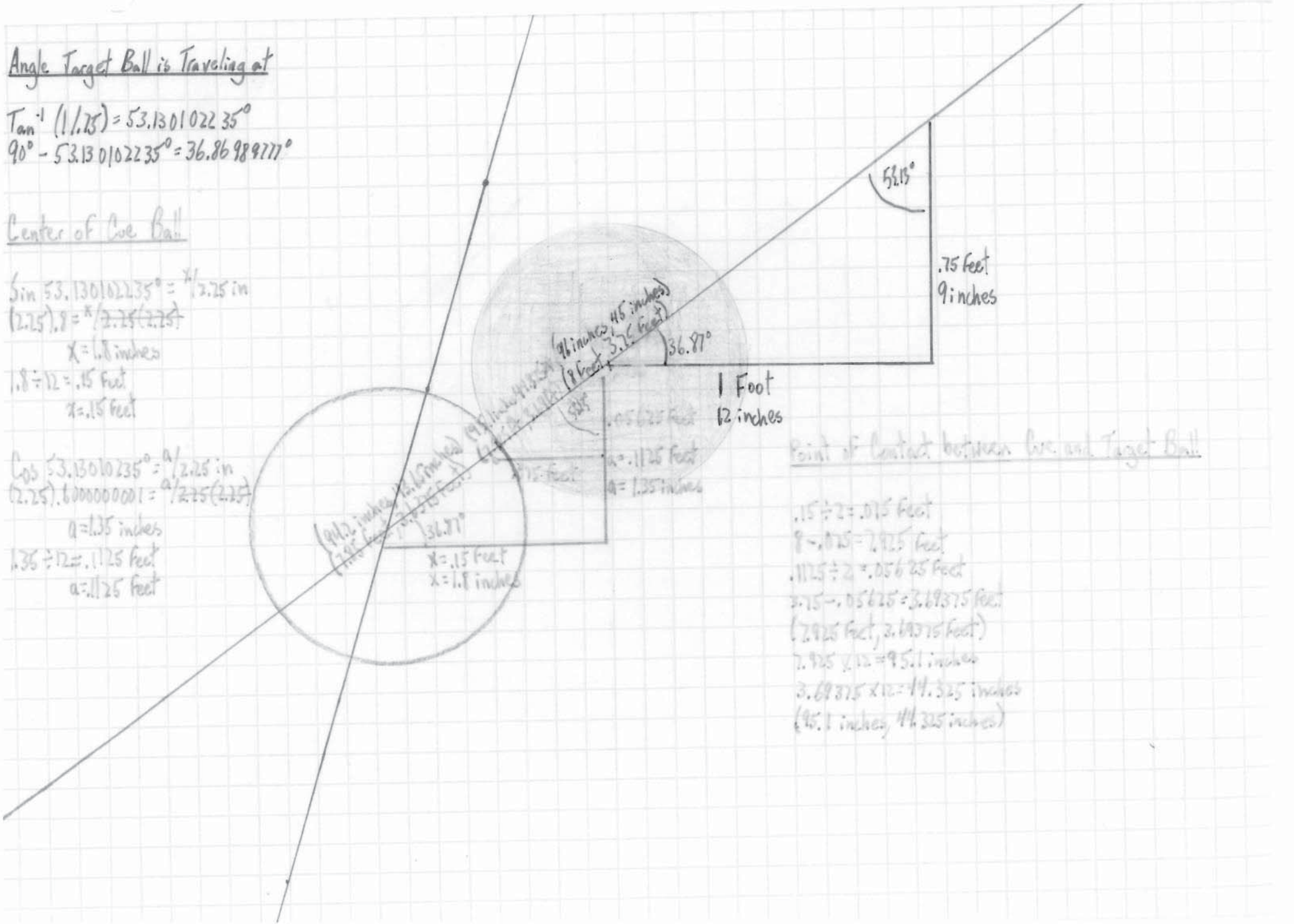
$\cos 53.13010235° = a/2.25$ in

$(2.25).600000001 = a/2.25(2.25)$

$a = 1.35$ inches

$1.35 \div 12 = .1125$ Feet

$a = .1125$ Feet



Point of Contact between Cue and Target Ball

$.15 \div 2 = .075$ Feet

$3 - .075 = 2.925$ Feet

$.1125 \div 2 = .05625$ Feet

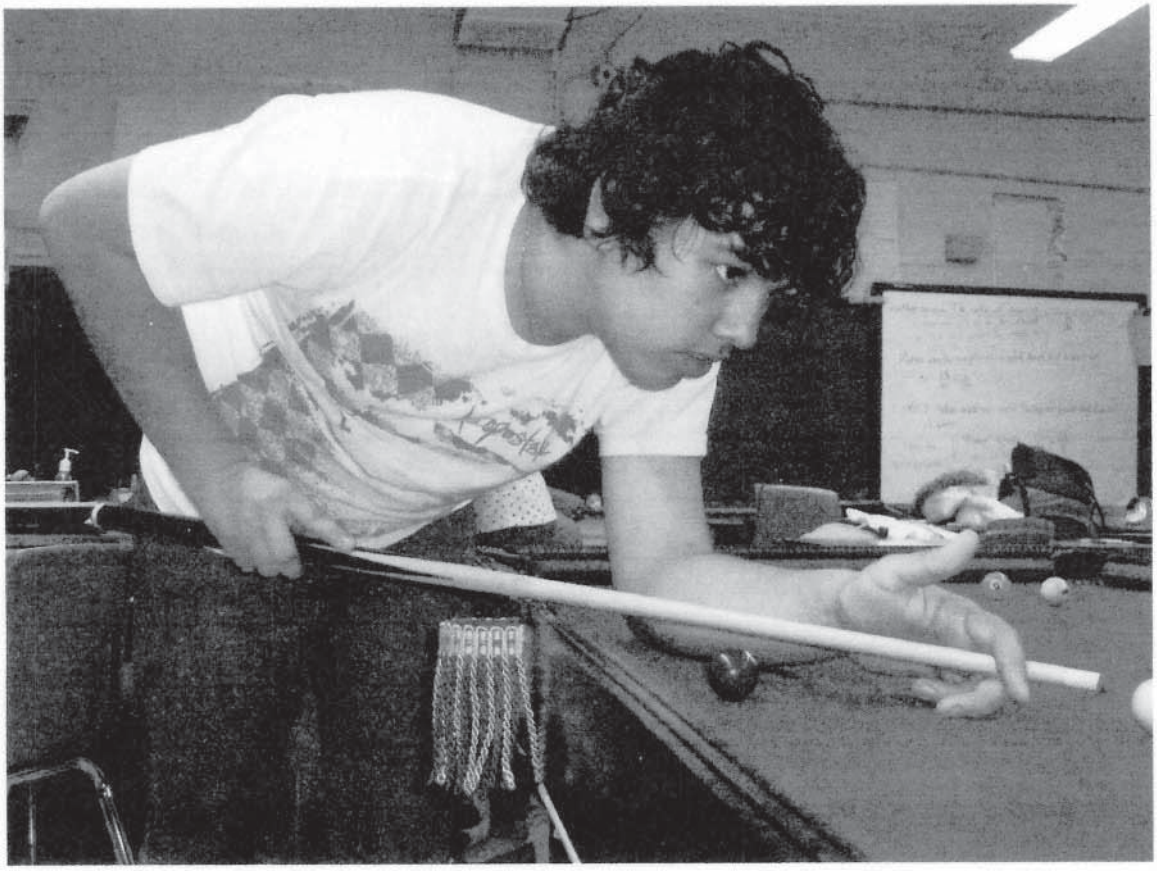
$3.75 - .05625 = 3.69375$ Feet

(2.925 Feet, 3.69375 Feet)

$2.925 \times 12 = 95.1$ inches

$3.69375 \times 12 = 44.325$ inches

(95.1 inches, 44.325 inches)



John Paul's Rail Shot

JOHN PAUL'S RAIL SHOT

ANGLE THAT TARGET BALL TRAVELS AT

TAN-1 1/2.5=
21.80140949=
21.8°
90 - 21.8° = **68.2°**

MEASURE OF SIDE A

SIN 21.8°=X/2.25
.3713678356*2.25=
.83557763=
.84 INCHES

MEASURE OF SIDE B

COS 21.8°=X/2.25
.9284858269*2.25=
2.08909311=
2.09 INCHES

CENTER OF CUE BALL AT CONTACT

(8, 2) FT
*12
(96, 24) IN
96-.84=95.16
24-2.09=21.9
(95.16, 21.9) IN
/12
(7.93, 1.825) FT

POINT OF CONTACT BETWEEN TARGET BALL AND CUE BALL

(8, 2) FT
*12
(96, 24) IN
.84/2=.42
2.09/2=1.045
96-.42=95.58
24-1.045=22.955
(95.58, 22.955) IN
/12
(7.965, 1.91) FT

MEASUREMENTS OF LARGE TRIANGLE

21.9+24=45.9 IN=HEIGHT OF LARGE TRIANGLE
95.16-72=23.16IN= BASE OF LARGE TRIANGLE

MEASUREMENTS OF SMALL TRIANGLE

21.9/45.9=X/23.16
507.204/45.9=45.9X/45.9=
X=11.05IN =BASE OF SMALL TRIANGLE

POINT AT WHICH THE CUE BALL HITS THE RAIL

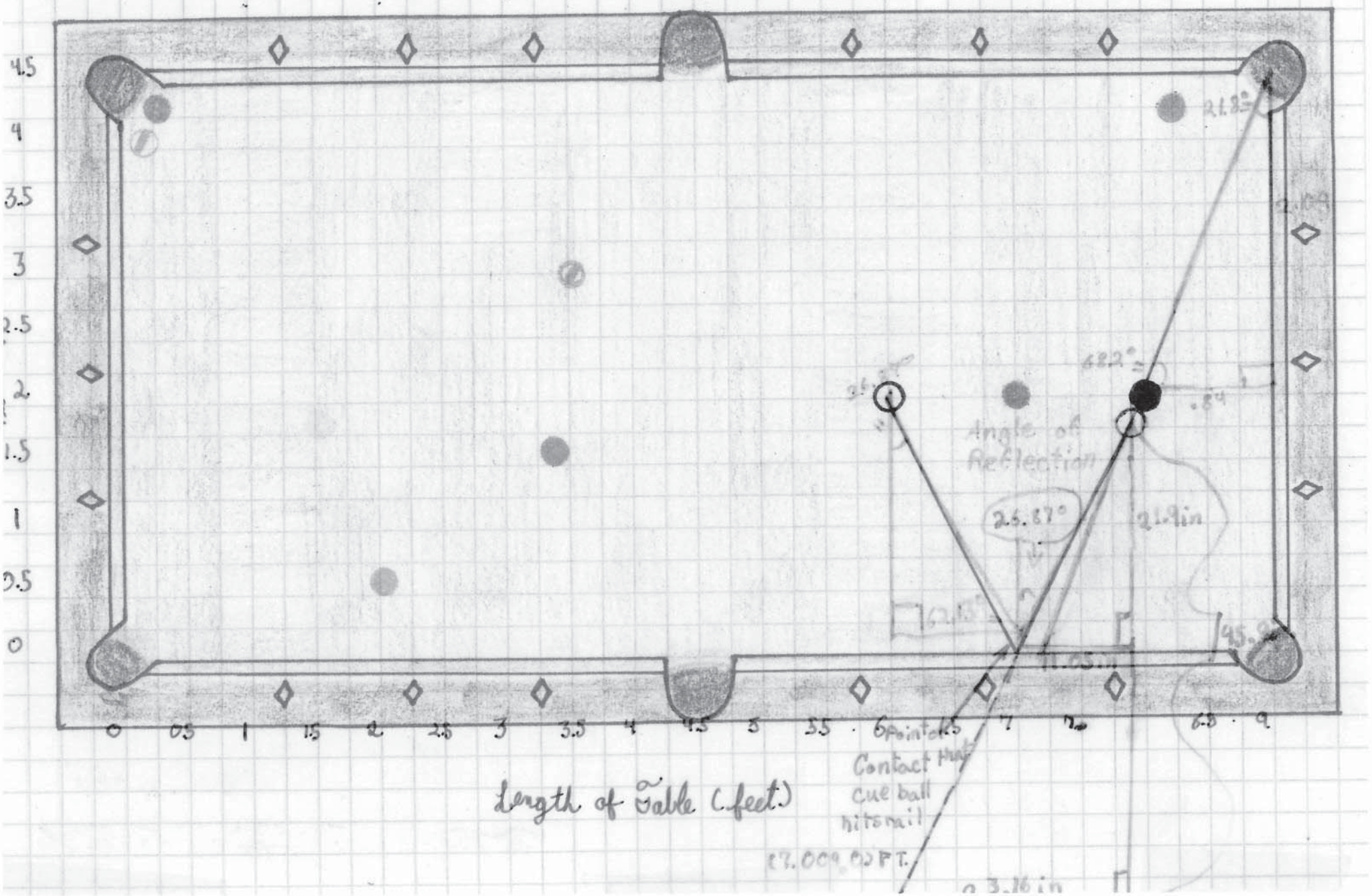
95.16-11.05=
(84.11, 0) IN
/12
(7.009, 0) FT

ANGLE OF REFLECTION

TAN-1(21.9/11.05)=
63.22603168=
63.23°
90°-63.23°=
26.77°

ANGLE THAT CUE BALL TRAVELS AT

63.23°+90°=
153.23°
180-153.23=
26.77°


4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
0 0.5 1 1.5 2 2.5 3 3.5 4 4.5 5 5.5 6 6.5 7 7.5 8 8.5 9
Length of Table (feet)
Angle of Reflection
25.67°
21.9in
42.2°
Contact Point
cue ball
hits rail
27.009 DPT.

Point on rail the Cue Ball is hitting

Big Triangle

21.9 in • 95.46 in = Height of Long triangle

95.46 - 23.16 in = 72.30 in = base of long Triangle

Small Triangle

21.9 in / 72.30 in = x / 11.05 in

72.30 in / 21.9 in = 11.05 in / x

x = 3.34 in base of small triangle

Point it should hit Rail like this

95.46 - 11.05 in

(84.41, 0) in

/12

(7.034, 0) Ft.

Angle of Reflection

$\tan^{-1}(21.9/11.05)$

$= 63.22603168$

$90° - 63.23° =$

$26.77°$

Angle That Cue Ball Travels At

$63.23° + 90° =$

$153.23°$

$180 - 153.23° =$

$26.77°$

*Work in orange on different transparencies

Eagle Target Ball is Shot at

$\tan^{-1}$ 1/2.5=

21.801409499=

21.8°

90°-21.8°=68.2°

Center of Cue Ball

in 21.8°= x/2.25

3713178356*2.25=

8355763=

.84 inches

cos 21.8°= x/2.25

92.84855 82.69*2.25=

2.089093111=

2.09 inches

(8,2) ft.

*12

(96,24) in.

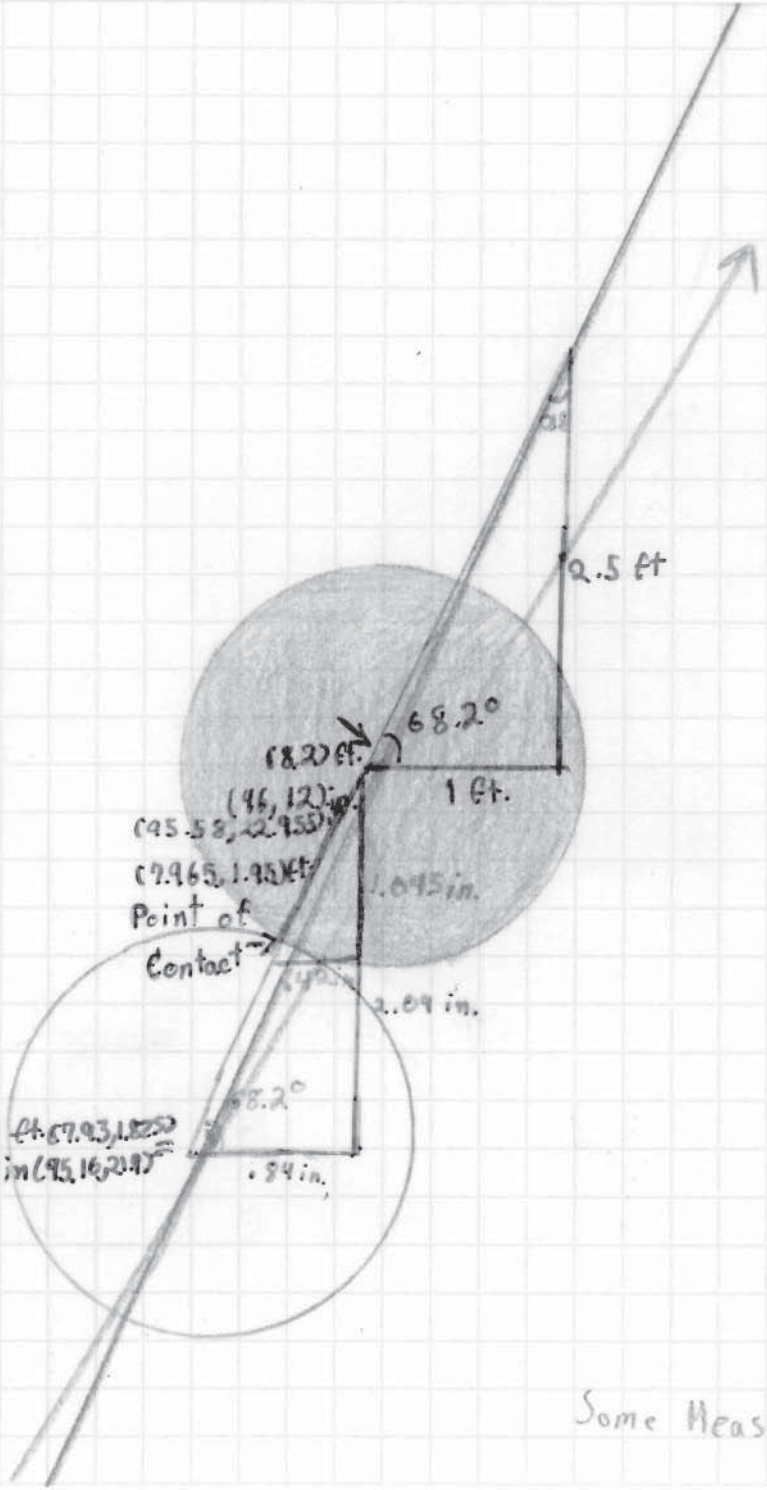
-.84=95.16

-2.09=21.9

(95.16, 21.91) in.

/12

(7.93, 1.825) ft.



Point of Contact Between Target Ball and Cue Ball

(8,2) *12

(96,24) in.

.84÷2=.42

2.09÷2=1.045

96-.42=95.58

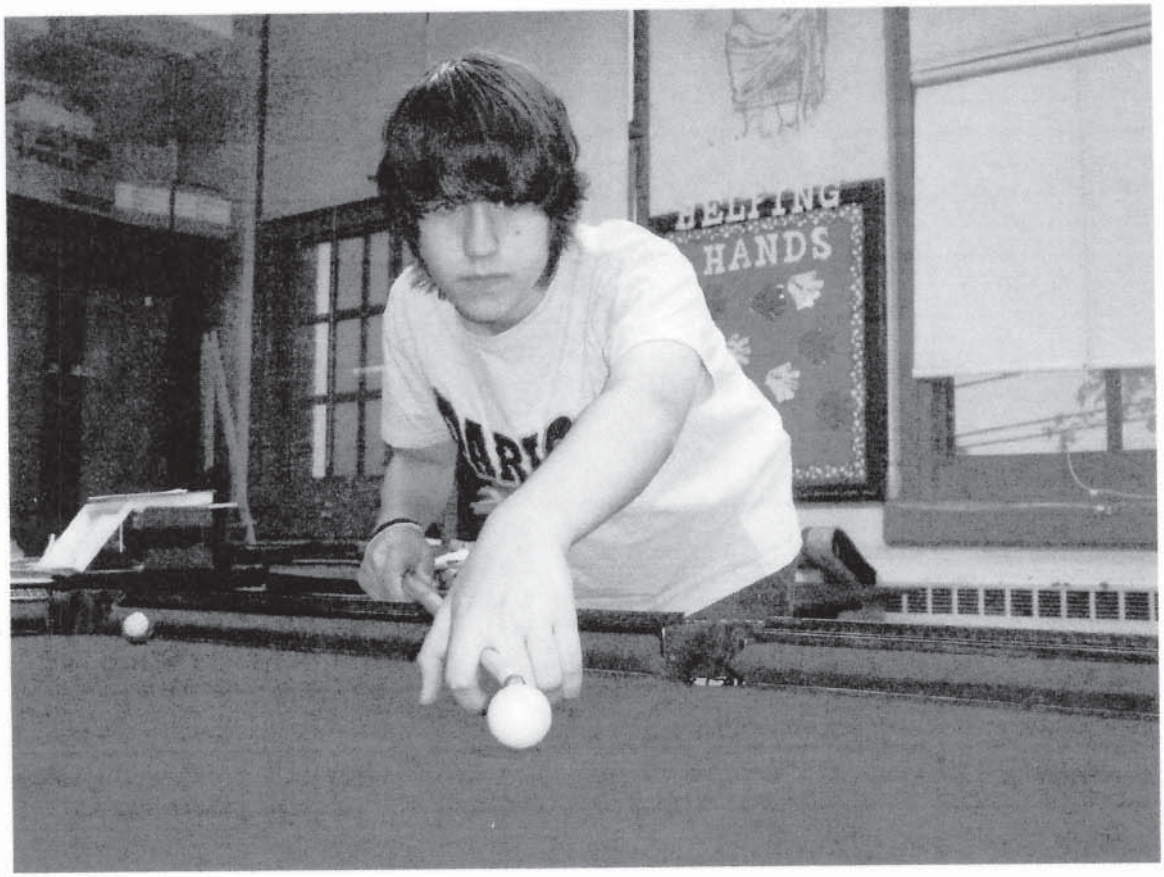
24-1.045=22.955

(95.58, 22.955) in

/12

(7.965, 1.91)

Some Measurements May Not be to Scale



Paul's Rail Shot

Paulo's Rail Shot

Angle at Which Target Ball is Traveling

$\tan^{-1}(2.5/3.75) = \mathbf{33.69^{\circ}}$

Center of Cue Ball at Contact

Measure of a

$\sin(33.69^{\circ}) = a/2.25$

$^{(2.25)}\ .5546992106 = x/2.25\ ^{(2.25)}$

$1.248073255 = a$

$1.25 \text{ inches} = a$

Measure of b

$\cos(33.69^{\circ}) = b/2.25$

$^{(2.25)}\ .8320509481 = y/2.25\ ^{(2.25)}$

$1.872114633 = b$

$1.87 \text{ inches} = b$

$(6.5, 3.75)\ ^{*12}$

(78, 45) inches

78-1.25= 76.75 inches

45+1.87= 46.87 inches

Inches-(76.75, 46.87) **Feet-(6.4, 3.91)**

76.75/12=6.39583

46.87/12=3.90583

Point of Contact between Cue ball and Target

(6.5, 3.75) *12

(78, 45)

1.25/2=.625 inches

1.87/2=.935 inches

78-.625=77.375

45+.935=45.935

Inches-(77.375, 44.935) **Feet-(6.45, 3.74)**

77.375/12=6.447916667

44.935/12=3.744583333

Point of Contact on Rail

4.5*12= 54 inches

54-46.87=7.13 inches

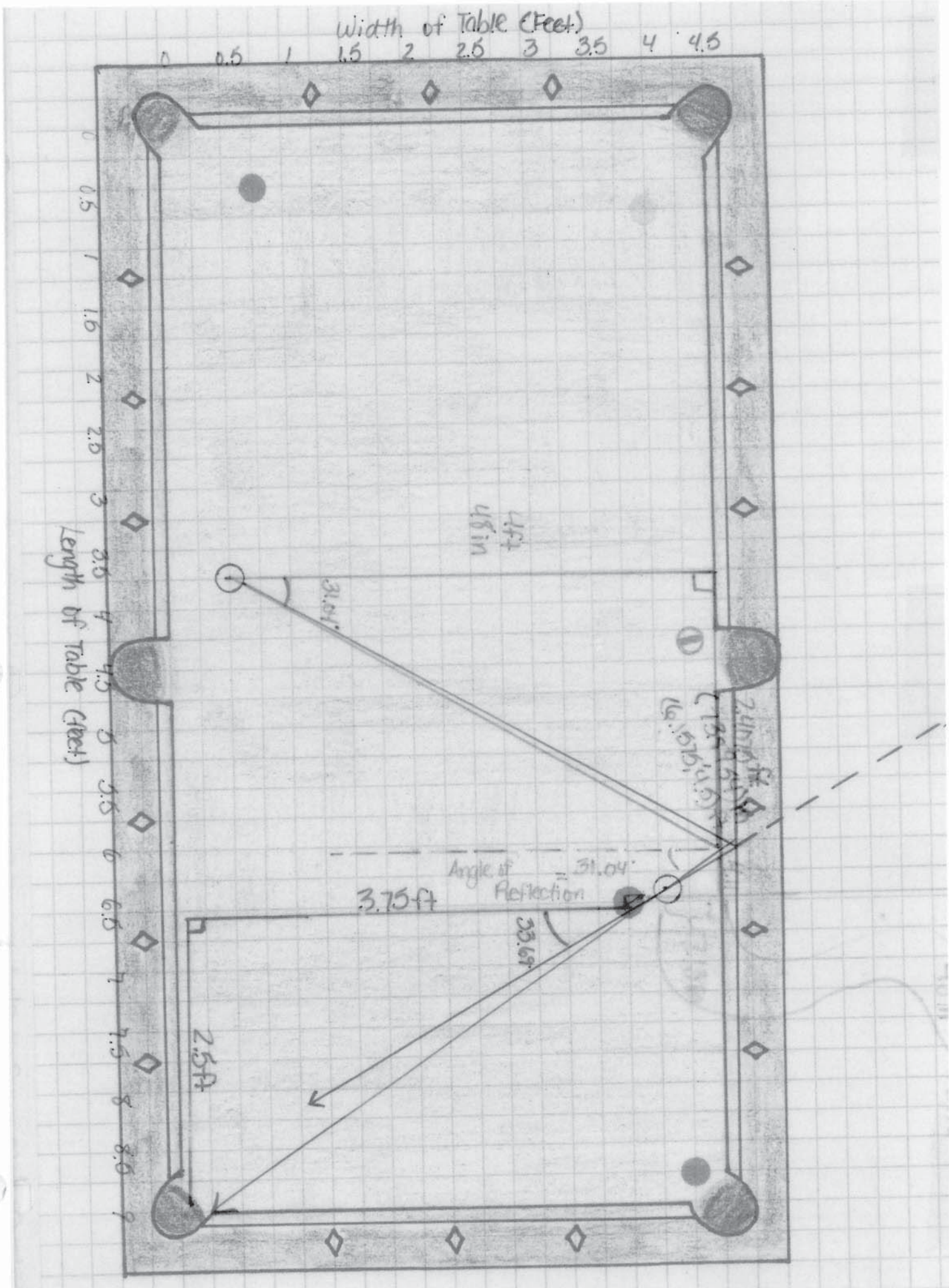
48 + 7.13= 55.13 inches

7.13/55.13=x/31.75
226.3775/55.13
4.106248866
≈4.11 inches
6.5*12=78
78-4.11=73.89
(73.89, 54 in)
(6.1575, 4.5 ft)

Angle Cue Ball is Struck at

6.1575-3.75= 2.4075 ft
Tan^{-1} (2.4075/4)=31.04268554
≈31.04°

Angle of Reflection
The angle of refection is equal to the angle the cue ball is struck at. The angle of reflection is **31.04°**


Width of Table (Feet)
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
Length of Table (Feet)
4ft
4.5in
31.04°
Angle of Reflection = 31.04°
3.75ft
33.69°
2.5ft

Point of Contact on Rail

4.5x12=54
54-46.87=7.13in
48+7.13=55.13in
7.13/55.13=x/31.75
226.3775/55.13
4.106248...
≈4.11inches
6.5x12=78
78-4.11=73.89
(73.89, 54inches)
(6.1575, 4.5 feet)
76.75-45=31.75
6.1575-3.75=2.4075

Angle Cue ball is struck at

$\tan^{-1}(2.4075/4ft) = 31.04268364$

≈31.04

Angle of Reflection

The angle of reflection is similar to the angle the cue ball is struck at. The angle of reflection is 31.04

* work in red on different transporencies

Angle at which target ball is traveling

$\tan^{-1}(2.5/3.75) = 33.69°$

Center of Cue Ball

$\sin(33.69°) = x/2.25$

$.554699216 = x/2.25$ (2.25)

$1.248073236 = x$

$\approx 1.25 \text{ inches} = x$

$\cos(33.69°) = y/2.25$

$.832114833 = y/2.25$ (2.25)

$1.872146325 = y$

$\approx 1.87 \text{ inches} = y$

$(1.25, 1.87)$ inches

Point of Contact between Cue ball and target

$(6.5, 3.75)^{\times 12}$

$(78, 45)$

$1.25 / 2 = .625$ inches

$1.87 / 2 = .935$ inches

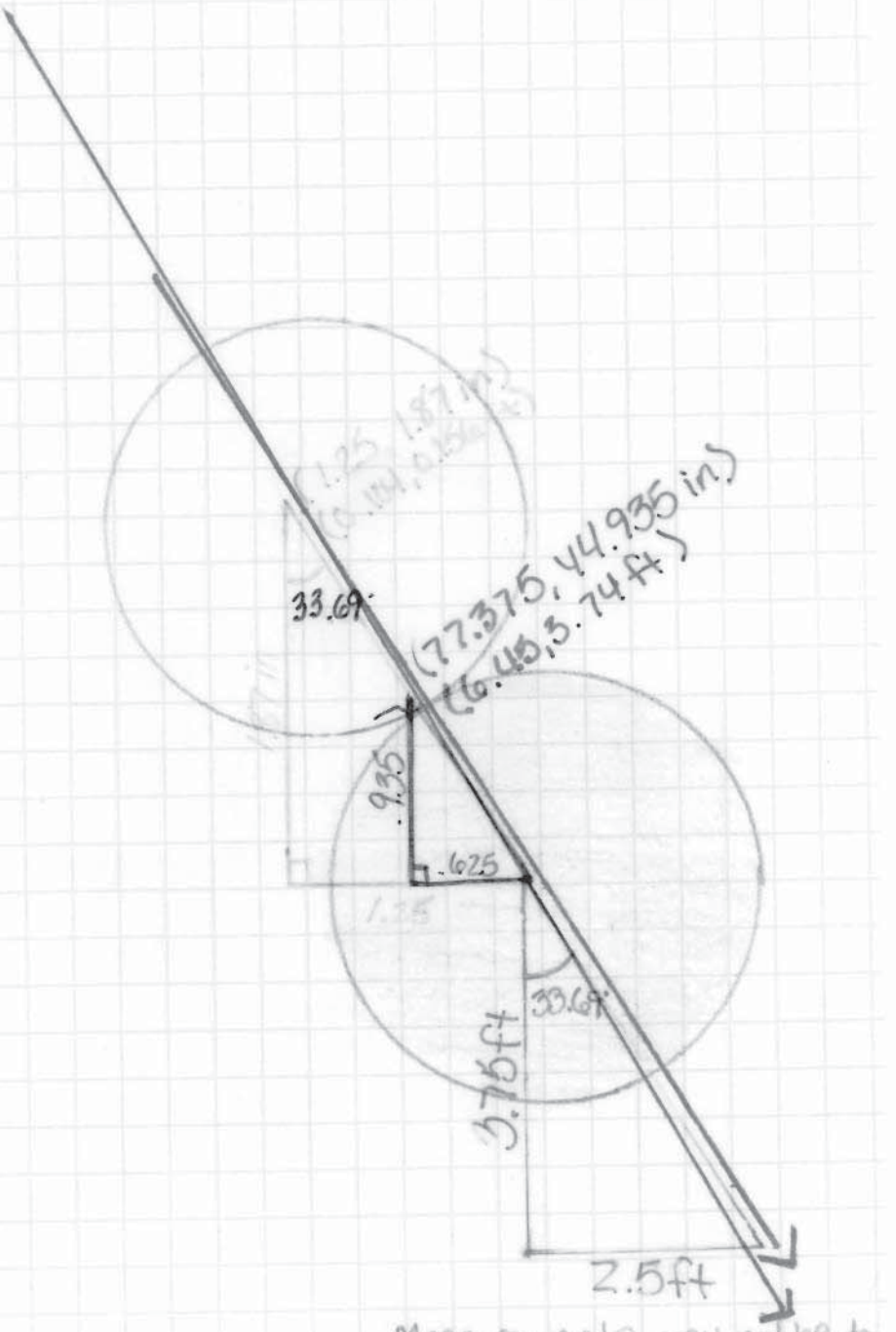
$78 - .625 = 77.375$

$45 + .935 = 45.935$

Inches - $(77.375, 44.935)$ Feet - $(6.45, 3.74)$

$77.385 / 12 = 6.447916667$

$44.935 / 12 = 3.744583333$



Measurements may not be to scale